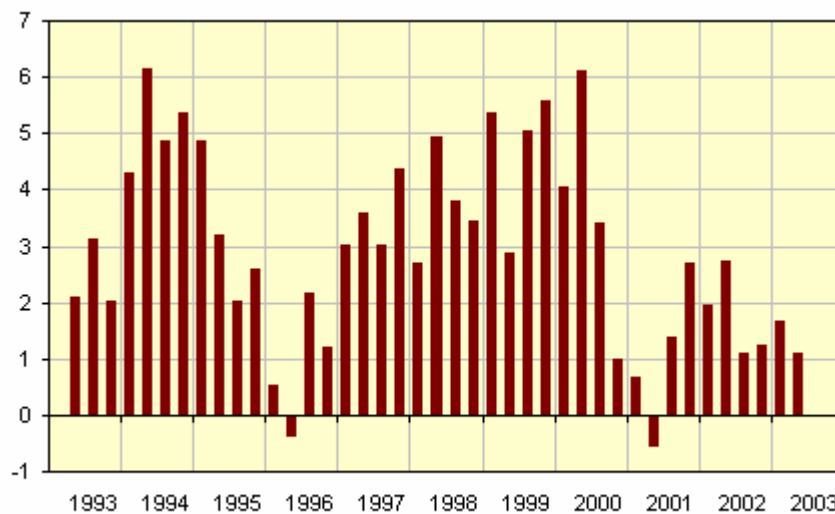


Seasonal Adjustment of National Accounts - An Overview -

Sven Öhlén

Gross domestic product (GDP)
 Per cent change from previous quarter, annual rate. Constant
 prices.
 Seasonally adjusted



Source: Statistics Sweden

Data up to and including the Second Quarter 2003

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1. BACKGROUND ¹

Seasonal adjustment has since long time been carried through with the program X-11-ARIMA on a mainframe at Statistics Sweden (SCB). After some methodological discussions in 1998, the time series function at the department of economic statistics organised a research program on seasonal adjustment (SA) at SCB. This research was made within the framework of a task force at Eurostat on SA, which started already in 1996. This working group discussed important issues on SA, e.g. the choice of software, direct or indirect SA, quality of SA, etc. Finally, some recommendations for SA among member states including choice of software were taken and published. Perhaps, the most important decision was the choice of software. The most important candidates were X-12-ARIMA (X12) and TRAMO/SEATS (TRS). X12 was a development of the X-11-ARIMA-program and TRS a new program permitting model-based SA, developed by A. Maravall and V. Gomez². Earlier experiences at SCB³ and the work made at Eurostat, the evaluation of software at SCB was restricted to the two programs X12 and TRS.

The evaluation at SCB has shown that SA with TRS applied to Swedish time series produces SA series with high quality. For that reason SCB has chosen to use TRS for SA of official time series. The implementation of TRS has been made for all national accounts series. The transition from X-11-ARIMA to TRS will be made for all time series at SCB

Although restricted resources, SCB has put much attention and effort to achieve and maintain high quality in SA of national accounts (NA) within the framework of model-based SA. Special attention has been made to calendar and/or working-day -effects and outliers. 136 time series were seasonally adjusted at the national accounts in 1999. Every single time series has been

¹ The author will thank Lena Hagman and Lars-Erik Öller, who have given constructive suggestions on the manuscript.

² Bank of Spain.

³ Structural time series models have been tested earlier at SCB. See (Öhlén, S. (1991).

treated individually. Many decisions have been taken to attain and maintain good quality in SA in order to make it easy for the user to analyse the Swedish economy and its development over time, the identification of turning points but also separate the effects of exogenous and random disturbances.

The methods now in use at NA are of high quality, as high as is technically possible. The recommendations now made by Eurostat have in most parts been in use at SCB since 1999 in regular production of SA of NA. This does not mean, of course, that ‘correct’ time comparisons can always be made for arbitrary points in time. Non-response and other errors do not vanish through efficient SA. As a matter of fact, efficient SA will illuminate different sources of errors, e.g. by outlier estimation. At last, we must stress that because the seasonal factor cannot be directly observed, i.e. a latent variable, there is no ‘correct’ method of SA. There is no ‘true’ SA graph. Every method and its application to real data is a compromise between many aspects and arguments.

The purpose of this overview of SA of Swedish NA is to give a very simple description of the principles taken for SA in order to produce statistics with high comparability over time.⁴ The types of sources of variation discussed in SA are introduced in chapter 2. *What is meant by a good method of SA* is a central question in chapter 3.1 followed by a short presentation of the research done at SCB in chapter 3.2. A brief outline of the ARIMA-world is given in chapter 4 with some important details for the understanding of the programs TRS, e.g. parameters, estimation, likelihood, etc. Chapter five treats very important choices in model-based SA, e.g. the choice of an ARIMA-model for the series and how these choices have been solved by SCB. The paper ends with some suggestions about the improvement of SA of NA at SCB followed by some illustrative graphs

2. SOURCES OF VARIATION OF TIME SERIES

There are a lot of different sources of variation in the observed values of time series. ‘Modern’ seasonal adjustment discriminates between six factors, the *trend*, the *cycle (C)*, the *seasonal (S)*, the *calendar effects (K)*, the *outliers (E)* and *irregular effects (I)* (random). Every observed value in a time series, O_t is the sum of these non-observable factors,

$$O_t = T_t + C_t + S_t + K_t + E_t + I_t \quad (2-1)$$

The seasonally adjusted and calendar adjusted series is given by subtracting the seasonal factor and the calendar effect from the original series, i.e.,

$$SA_t = O_t - S_t - K_t = T_t + C_t + E_t + I_t \quad , \quad (2-2)$$

⁴ A statistical methodological report and a web-version will be published during 2004 in English.

consisting of the ‘trend cycle’, the effects of outliers and irregulars. For now on, we use the notation SA for a seasonally adjusted and calendar adjusted series (including the number of working-days).

In many NA series, the importance of the irregular is not important. On the other hand, the outliers E_t may have a great impact of the series. The reason to include outliers as separate components is that the calculation of the seasonal factor will become more precise. The trend is the long-time development of the process, e.g. economic growth. The cyclical component is supposed to depend on such cyclical phenomena as the ‘business cycle’.⁵ The distinction between trend and cycle is sometimes not explicitly given but put together to the common component trend-cycle. This approach will be taken in this paper.

Climate, institutional factors and/or the behaviour of consumers are often the causes of seasonal effects, e.g. consumption at Christmas Eve. In Sweden, there is a very unusual seasonal effect depending on industry holidays in every July. By calendar effects we mean any effect stemming from the calendar, e.g. the number of working-days in a period, the composition of days during a quarter, the number of Mondays, Tuesdays, etc. The length of a period is also treated as a calendar effect as well as the Eastern effect.

There are many important requirements for the adequacy of the model (2-1) and for an appropriate estimation of the components. One of these is the assumption that the components of (2-1) are independent. If e.g. the cyclical factor depends on the seasonal factor or other factors, the foundation for quality SA has decreased. Another assumption is that the distribution of the irregular factor is normal and not changing over time. For instance, if the variance of this distribution is increasing, we have to make some adjustments to make proper estimation of the model and its parameters. Such statistical requirements are more and less fulfilled with real data. Most of the statistical assumptions for a specific model and real data can be checked by statistical tests and diagnostics in ‘modern’ SA.

(2-1) Is called a model for a time series with additive components (additive model). The majority of time series, the components are not additively related to the observed value. Instead there are multiplicative effects, e.g., the seasonal variation depends on the level of the series. In such cases, we use a multiplicative model according to

$$O_t = T_t C_t S_t K_t E_t I_t \quad (2-3)$$

It is obvious that if we take the logarithm of (2-34), we get the additive model. Consequently, SA could be discussed in terms of an additive model according to (2-1).

The reason to perform SA is to make it possible to compare any observed value of the series to another value. If such comparisons are made on the original

⁵ However, it is not precisely equivalent to the business cycle in economic sense.

series, any measure of change will depend on the components of the series according to (2-1). By estimating every component and how they developed over time, we can eliminate such ‘not interesting’ variation. However, what and what is not interesting or important depends on the purpose of use. For most users the actual estimate of the seasonal factor is not important⁶. The relative importance of the factors of (2-1)⁷ is different for different series. Furthermore, it varies over time for a given time series. For most series, the seasonal factor is the most important factor. However, the effects of ‘exogenous’ factors such as ‘the number of working-days’ may have significant effects. All these components are estimated in what we call seasonal adjustment. In the next chapter, a short discussion is given on the issue of choice of statistical method of SA for the national accounts at SCB.

3. RESEARCH ON SEASONAL ADJUSTMENT AT SCB

Seasonal adjustment is a rather complex process. Many decisions have to be made in order to maintain good quality. ‘What is good quality?’ in SA and ‘how can it be maintained?’ are the main issues in this chapter. The research in SA at Eurostat, ECB and SCB is also briefly summarized.

3.1 What is good Quality in Seasonal Adjustment?

A central issue has been the characteristics of good quality in SA. The general recommendations made by Eurostat have been taken as a good starting point although an exhaustive international discussion of the recommendations does not seem to have taken place. These recommendations are described below⁸.

a) Theoretical properties

Probably the most important guideline contains the property ‘scientific method’ or ‘statistical method’. By this we mean that in model-based SA, the specification of the model for the series should be made on statistical principles, e.g. the likelihood-principle. It should be possible to check in a scientific way, the assumptions made by means of statistical tests. That includes all assumptions of the model, e.g. the distribution of the residuals, assumptions of independence of the components, etc. The model should be consistent with data. The estimation of the parameters should also be made on statistical accepted principles in order to make statistical inference from estimates to the corresponding true parameters. If so, it will be possible to calculate measures of uncertainty for the parameters of the model and also for the components of the decomposition. It will also be possible to calculate and publish measures of uncertainty like confidence intervals. The SA method should also have sufficient diagnostics in order for the validation of the procedures.

⁶ For salesman of ice-cream etc, seasonal variation is very important for retail sales..

⁷ In relation to the observed series.

⁸ See http://europa.eu.int/en/comm/eurostat/research/noris4/documents/plcy_v40.ppt

b) Empirical properties

The model for the series should have high goodness of fit to data in statistical sense.⁹ If a SA series is adjusted once more, the new SA series should be the same (idempotent). Yearly totals of the SA series and the original series should be equal (time consistency). Many users of SA series would prefer the concept of consistency in aggregations of SA series. For instance, SA GDP and the components of GDP should be equal. A SA series with low variability should be preferred instead of SA series with high variability, if they are equal in other quality aspects.¹⁰ For SA methods relying on forecasts the forecast errors should be as small as possible.¹¹ The revisions due to SA should also be small¹². SA series should signal turning points 'as soon as possible' with a low rate of false alarm¹³. Any SA method uses filter to estimate the components of the series. Such filters should be optimal and the extraction of signals of the model should be efficient and adapt to data. At last, the method should have a software user-interface, e.g. in Windows.

c) Other aspects

SCB has investigated other aspects of SA methods as the statistical property *bias*. By bias we mean the average deviation between the true value of a component and the estimated value. The properties of the software have been discussed. How is the statistical methods programmed? How is the module for input/output? How is the software designed for mass-production of SA series? Could the software be easily implemented on the platforms used by SCB, etc.¹⁴? The issue about direct and indirect SA has been discussed within the framework of the task force for SA at Eurostat. This issue is still not solved. It will be further discussed in chapter 5.10.

3.2 Research at SCB

It is quite obvious that SA has many dimensions and the lack of complete consensus on SA matters is not surprising. Many desirable properties, not even consistent, have been formulated. The final choices for the SA of a time series are compromises between what is desirable and what can be achieved. In this paragraph, we give a short review of the research on the choice of software carried through at Eurostat and SCB. The tests of the DOS-versions of the programs TRAMO/SEATS and X-12-ARIMA started late 1998. There are user manuals for the programs, describing the installation and also the input parameters of the programs. Free format text-files are used for input. In 1998

⁹ One example of this is that the model is selected by the principle of maximum likelihood given the data.

¹⁰ This is not a generally accepted property but have been accepted at SCB.

¹¹ This is of course a 'reward' due to a proper model for the actual series.

¹² By this, we do not mean the revisions of the original series.

¹³ This user requirement cannot be fulfilled because an early signal has greater uncertainty as compared to a late signal

¹⁴ See Skölleremo and Öhlén (2000). In Swedish.

there was no interface to Windows with the exception of TRS. Eurostat had developed an EXCEL-macro^{15 16}. In the report Lundqvist, P. and S. Öhlén (1998) tests of bias in the estimation of the components of a time series model were made. The choices of the series, e.g. the seasonal and the irregular variability were made to give similar properties as actual published deliveries series at Statistics Sweden. The report was presented at the SAM98-conference in Bucharest in Oct. 1998. 1000 series were generated with known systematic components. The properties bias, RMSE, correlation between the estimated and true components was investigated. There was no significant differences between the programs. They perform quite well in terms of bias and RMSE. However, it was found that the special typical Swedish seasonal pattern¹⁷ caused estimation problems for both programs. In Öhlén (1998:3), an empirical study from 66 national accounts was made regarding

- The automatic model-selection module,
- Time consistency
- Smoothness,
- Calendar effects,
- The identification and estimation of outliers.

In general X12 will identify a more complex ARIMA-model. Only in rare cases, the same ARIMA-model was identified. Time consistency was not a big problem for the different softwares. TRS produces SA series with less variability. The treatment of calendar effects was quite different. In 13 cases, X12 finds an effect from Easter – TRS only in one case! The corresponding for outliers are 27 respectively 17.

The estimation of outliers and calendar effects were further studied in Öhlén, S. (1999:1) using a fix ARIMA-model. 13 quarterly series from national accounts were used. SCB has since long used a working-day adjustment with the ad hoc ratio method. The merits of this method was tested against the regression method with the two programs. X12 and TRS produced similar working-day effects, which were quite different from the corresponding ratio-method used by SCB.

The ratio-method did not work well. It was concluded that the ratio-method produced an over-compensation of the working-day effect. The internal trading-day variables of the programs were also tested. There were remarkable differences between the programs. The estimation of the ARIMA-models were in most cases 'acceptable'. A further investigation of the calendar estimation provided by the programs was carried through in Öhlén, S. (1999:2) by simulations using a known calendar model. X12 provided correct estimates, TRS not. This report was sent to A. Maravall and the bugs in the program were verified and corrected for versions including TRS June 1998 and after. In Öhlén, S. (1999:2), it was concluded that the estimation of a fixed ARIMA-

¹⁵ This macro has been used at SCB on a limited scale. Some changes of the source-code have been made in order to use Office 2000. Regression variables cannot be used. There is no support from Eurostat for the macro.

¹⁶ X-12-ARIMA is implemented in SAS version 8, unfortunate with limited functionality

¹⁷ There is a very low value every July.

model with regression variables were not equal. In order to check the estimation of the ARIMA-model further, simulations were made in Öhlén, S. (1999:3) with known ARIMA-models. It was concluded that the estimation of the ARIMA-models was acceptable in both programs. The software SAS and AUTOBOX was used as a benchmark.

When new observations are included in a time series, there will be revisions of the SA series and the trend estimates. This revision problem was studied in Öhlén, S. (1999:4) for national accounts and short-term industrial indicators. It was concluded that revisions errors in terms of RMSE¹⁸ was lower with TRS as compared to X12. This was true for a given ARIMA-model but also when the programs used the automatic model-choice algorithm. These results were undisputed. TRS produces lower revision errors.

3.3 Research at Eurostat and ECB

Eurostat has since 1996 published many reports on SA issues including the choice of software¹⁹ some of which were presented at the SAM98-conference. As a result from these studies, Eurostat recommends model-based SA within the framework of TRS. However, X12 can be used as an alternative²⁰. ECB has come up with the recommendation to use X12 for ECB and the central banks of EU. However, TRS is regarded as the best method from a methodological point of view. Some arguments against TRS in terms of complexity and requirements put on the analyst were raised. A version of X12 with model-based decomposition in lines with TRS is in progress. This, maybe, will be a final choice SA software²¹.

3.4 The Choice of Method for Seasonal Adjustment

The research made at SCB has shown that the software TRS should be used for SA of time series at SCB. The report from ECB does not change that conclusion. In 1999 a user-interface between the DOS-programs of TRS was made for the national accounts series with SAS macro-language and EXCEL at SCB.

4. THE PROGRAMS TRAMO/SEATS

TRAMO (**T**ime Series **R**egression with **A**rima Noise, **M**issing **O**bservations and **O**utliers) is a Fortran-program under MS-DOS. It is run on PC and Windows with the software SAS. The program is used as the first step for seasonal adjustment with the program SEATS (**S**ignal **E**xtraction in **A**rima **T**ime **S**eries). In the program SEATS a decomposition of the series produced

¹⁸ Root mean square error.

¹⁹ <http://forum.europa.eu.int/Public/irc/dsis/eurosam/information>

²⁰ See footnote 23.

²¹ These questions and the recommendations given by Eurostat and ECB have been discussed at a CMFB-meeting in Jan. 2002 (Committee on monetary, financial and balance of payments statistics).

by TRAMO according to model (2-1) or (2-3) is made. In the program TRAMO different types of calendar effects, e.g. an Easter effect and different types of trading day effects are estimated. Different effects from outliers are also estimated by the program, which will have negative impacts on the quality of the seasonal adjustment, if not taken care of. There are many types of outliers, which are further discussed later in this chapter. TRAMO is also used to estimate the effects from user defined regression variables. In this particular case, the number of working-days are used.

4.1 What is an ARIMA-model?

The program TRAMO, SEATS but even the 'old' method X-11-ARIMA, which still is in use at SCB, is based on ARIMA-models. This is done in TRAMO integrated with the estimation of outliers and other deterministic effects and also in the decomposition of a series into a seasonal and other factors. In the next chapter, a short introduction to ARIMA models are given for the not informed.

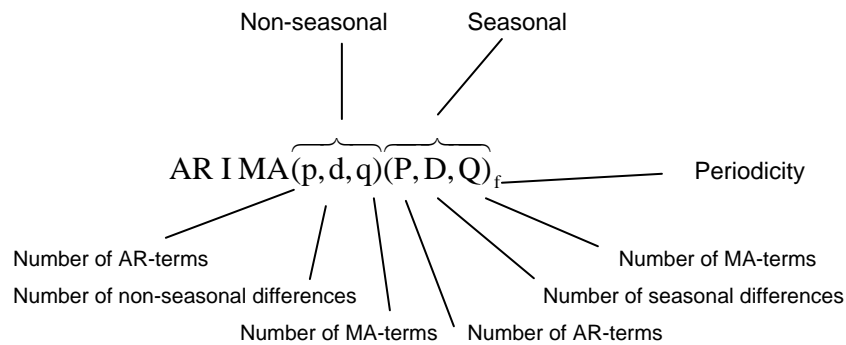
4.1.1 Notations of an ARIMA-model

The notation ARIMA is an abbreviation for Auto Regressive Integrated Moving Average and $(p, d, q)(P, D, Q)$ is a compact way representing the model structure. The notation SARIMA is sometimes used for seasonal ARIMA models. In the following we use the notation ARIMA even in the case of a seasonal model. The period of a seasonal model is usually a month or a quarter.

ARIMA-models are a very general class of linear time series. In a very famous theorem Herman Wold proved the existence of moving average process if it is stationary. The most important contribution to ARIMA-modelling was made by Box and Jenkins in 1970, which developed a general method to identify, estimate and forecast a time series represented by an ARIMA-model.²²

²² See Box, Jenkins (1970).

Graph 4.1.1 Notation of an ARIMA-model



First we introduce the elementary concepts, *time series, realisation, stationary and autocorrelation*. The AR(1) model for GDP is shown. Stationary is a very important property of a time series and ARIMA-modelling. We show how to transform a non-stationary series into a stationary series. Then a MA(1)-model is shown.

What is a time series?

Most of the users of statistics have a very clear opinion about the meaning of a time series. For some users, a time series is just numbers on a time scale. However, from a statistical point of view, it is very important to focus on the concept of probability. A time series Y_t is the outcome of a random variable indexed by time or other variable. It is very important to see Y_t as outcomes depending on a probability distribution. If the outcomes of the random variable Y_t are observed at times $\{t = t_1 < t_2 < \dots < t_T\}$, we usually write the outcomes

$$\{y_t, t = t_1, t_2, \dots, t_T\} \tag{4-1-0}$$

Instead of $\{Y_t\}$, we write Y_t or just y_t understood that it could be the stochastic variable of interest, its outcome for a subset of the time variable. If we know the distribution $P(\cdot)$ for Y_t , we can calculate the probability for different events, e.g. $P(Y_t < 0)$ or $Y_t < Y_{t-1}$. The probability of a certain outcome of the process is called the likelihood L for the series and has the notation $L\{y_t, t = t_1, t_2, \dots, t_T\}$. Usually the time points are equal spaced (equidistant) with the notation $t = 1, 2, \dots, T$ where T is the last observation. If a model represents the time series, the likelihood of the series depends on one parameter φ or several unknown parameters $\Theta = [\varphi_1, \varphi_2, \dots, \varphi_n]$. The model is the mathematical expression of the parameters and y_t . In order to calculate the probability of a certain time series given a model, it is necessary to know the probability for all points of time $t = 1, 2, \dots, T$, i.e. the multivariate distribution. Formally the probability of the series can be written

$L(\{Y_t, t = t_1, t_2, \dots, t_T\} | \Theta)$. The parameters Θ are unknown and must be estimated for a specific model. Different models normally have different parameters and different estimates of the parameters as well as different time span produces different likelihood for the observed series. The estimation of the parameter Θ in TRAMO/SEATS is based on the principle of maximum likelihood. It can be formulated as follows.

The principle of maximum likelihood²³ (ML): Given a time series $y_t, t = t_1, t_2, \dots, t_T$ and a model of the series depending on parameters Θ . Estimate Θ such that the estimate $\hat{\Theta}$ maximizes the likelihood

$$L(\{Y_t, t = t_1, t_2, \dots, t_T\} | \hat{\Theta}).$$

That means that any other linear estimate of Θ gives a less satisfactory description of the time series. The principle of ML is used in TRAMO/SEATS and for normal distributions, it has been proven to produce estimates with optimal properties in statistical sense.

Stationary

Stationary is a statistical property that is very important in time series analyses, e.g. in ARIMA-modelling and forecasts. If the probability distribution $P(.)$ for Y_t does not change over time, the time series is said to be stationary. Usually economic time series have a trend and the expected value of Y_t is changing over time. Usually the variance of Y_t is not stable but depends on time. The first moment in ARIMA-modelling is to check if the time series is stationary. If not, different transformations are made to put the series into a stationary form. A common trick is to eliminate a linear trend by using differences (described in the next section). In TRAMO/SEATS a logarithmic transformation is used.

Autocorrelation

In economic time series there is usual some sort of dependency between the outcomes for certain lags. The correlation coefficient between Y_t and Y_{t-h} is called the autocorrelation, $\rho(h)$ for lag h and is defined as

$$\rho(h) = \frac{E[Y_t - E(Y_t)][Y_{t-h} - E(Y_{t-h})]}{\sqrt{E[Y_t - E(Y_t)]^2 E[Y_{t-h} - E(Y_{t-h})]^2}} \quad (4-1-1)$$

where E is the expectation operator. Vi can calculate (4-1-1) for a certain time series. We can also use the autocorrelation and similar statistical tools to get information on which type of ARIMA-model has produced a certain time

²³ This principle is perhaps the most used and accepted principle in statistics.

series²⁴. It is a very important tool to describe a time series model but also to check if the residuals given by a certain model follows what we call ‘white noise’. Autocorrelations of the residuals from seasonal adjustment is produced and analysed for all national accounts series.

BIC

A very important step in model based seasonal adjustment is to search for a proper ARIMA-model for the series. Because the class of ARIMA-models is very large, this can be very time consuming. There are many statistical tools, which can be used in the search and selection of a proper ARIMA-model. The most important is called “Bays Information Criterion”, BIC defined as

$$\text{BIC} = -2 \log(L) + K \log(T),$$

where K is the number of parameters and T the number of observations in the series. The inclusion of more parameters in a model usually results in higher likelihood in some cases to many (overparametrization). BIC gives a penalty for overparametrization. A similar measure is Akaike's Information Criterion(AIC), and defined as

$$\text{AIC} = -2 \log(L) + 2 K$$

The model with $\min(\text{AIC})$, $\min(\text{BIC})$ is best according to the AIC and the BIC criterion respectively. Further discussions of these criteria are given in Parzen (1974), Schwartz (1978) and Schibata (1976,1986). A few examples of ARIMA-models are given in the next chapter.

4.1.2 Autoregressive models

Many economic time series $\{Y_t\}$ can be approximated by the model

$$\sum_{i=0}^p a_i Y_{t-i} = e_t, \quad (4-1-2)$$

where a_i are parameters (fixed numbers) and e_t is an uncorrelated random variable. This is an autoregressive model of order p , abbreviated AR(p) where p is the order of the autoregressive structure (the number of terms with time dependency). A very common type is the AR(1) model where Y_t depends only on Y_{t-1} , i.e.

$$Y_t = aY_{t-1} + e_t \quad (4-1-3)$$

²⁴ This moment is usually called the *identification*. A classical reference is Box & Jenkins (1970).

By using the recursiveness

$$Y_{t-i} = aY_{t-i-1} + e_{t-i}$$

for $i=1, \dots, T$, the model (4-1-2) can be expressed as

$$Y_t = a^T Y_{t-T} + \sum_{i=0}^{T-1} a^i e_i$$

If $|a| < 1$ then $a^T Y_{t-T}$ will be small for large T , and an AR(1) can be written as²⁵

$$Y_t = \sum_{i=0}^{\infty} a^i e_i \quad (4-1-4)$$

That is, an AR(1) and a sufficient long time series can be represented by the outcomes of a series of random variables. This is an example of different representations of ARIMA-models because (4-1-4) is a so-called moving average process. There are statistically equivalent representations of a given time series even if they seem to be parametrically different. In (4-1-2) the outcome of Y_t depends on the outcome of Y_{t-i} and the parameters a_i . The autocorrelation coefficient between Y_t and Y_{t-i} for an AR(1) model can be shown to be $\rho(i) = a^i$. It depends on the autoregressive parameter a and the time distance between the outcomes. Now, we expect that the absolute value $|a| < 1$. Thus, $\rho(i)$ is a decreasing function in the time distance between the outcomes and close to zero for distant observations. In similar ways, the mathematical expression of the model can be used to derive dynamic properties of the series (model). We can also go the other way around. From knowledge of the estimated dynamics of a particular series, we can get information of the unknown model, which has produced the series. This can be used in the identification of the model for a particular series. Now, we will give some examples of an AR(1) model for GDP.²⁶

Suppose GDP for a quarter t depends on GDP the quarter before²⁷, it can be written

$$GDP_t = m + aGDP_{t-1} + e_t \quad (4-1-5)$$

where m, a are parameters and e_t is a normally and independent distributed random variable with zero mean and standard deviation σ (white noise). If this model is estimated for data covering 1993-2001 in prices of 1995, we get the following estimated AR(1) model

²⁵ If e.g. $p=9$, $X=100$ and $a=0.50$, the expression will be 0.10.

²⁶ It can also be written ARIMA(100)(000).

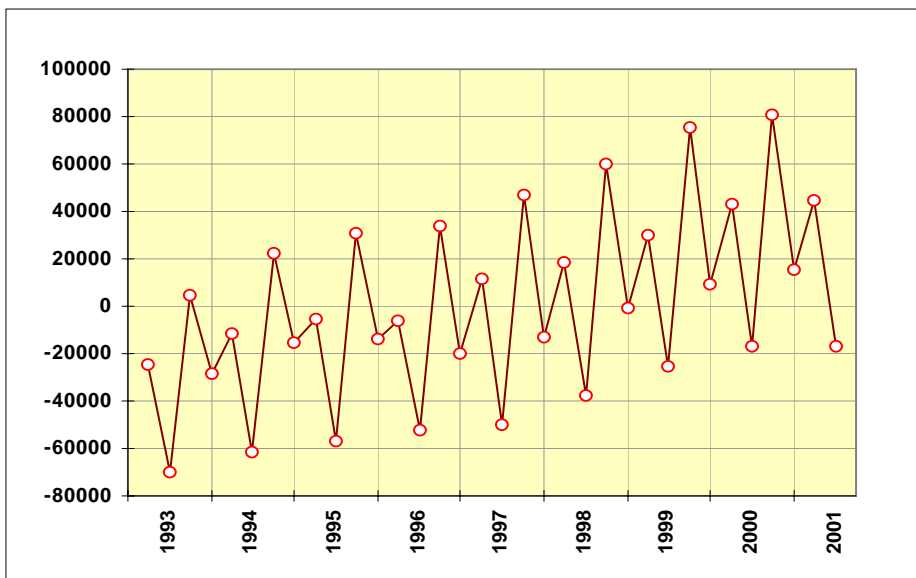
²⁷ This model is also called a Markov process with drift.

$$GDP_t = 71325 + 0.392GDP_{t-1} + e_t \quad (4-1-6)$$

(3.8) (2.5)

AIC=817, $R^2 = 0.13$, $\sigma = 38926$ and the number within brackets is a t-test of the hypothesis that the true parameter value is zero. The t-test of $a=0$ is rejected at the significance level 5%. R^2 is the multiple correlation coefficient indicating the fit. $R^2 = 1$ indicates a perfect fit and $R^2 = 0$ indicates no fit. Although the model is significant, the fit of the model is low with $R^2 = 0.13$. This low fit suggests that model (4-1-6) is not proper. What are the causes for the poor fit? Let us examine the residuals from this estimated model for GDP, i.e., the estimated series e_t . The residuals are shown in graph 4.1.1

Graph 4.1.1 Residuals from model (4-1-6) for GDP



It appears that the residuals have a trend and also a seasonal pattern, i.e. they are not white noise. The mean of the residuals depends on time and there is also autocorrelation. The autocorrelation of the residuals is shown in table 4.1.1 for different lag. Lag=1 is short for quarter t-1, Lag=2 quarter t-2, etc. In the table a t-test of the hypothesis that the autocorrelation is zero is shown for different lags. It is significant for Lag=2 and Lag=4. We must remove this seasonal variation of the residual and also the trend from the series to produce a better residual. The most common way to do so is to differentiate at the seasonal period. This can be written²⁸ $(1 - B^4)GDP_t = GDP_t - GDP_{t-4}$. Instead of modelling the GDP-series, the differenced series is used.

Table 4-1-1 Autocorrelations of the residuals for model (4-1-6)

Lag	Autocorrelation	Standard-	t-test
-----	-----------------	-----------	--------

²⁸ The operator B^k is defined as $B^k(y_t) = y_{t-k}$.

	coefficient	deviation	
1	-0.26	0.17	-1.53
2	0.53	0.18	2.94
3	-0.30	0.22	1.36
4	0.81	0.23	3.52
5	-0.31	0.31	1.00
6	0.38	0.32	1.19
7	-0.32	0.33	0.97
8	0.63	0.34	1.85

Instead of modelling the original series of GDP, we model the seasonal differences by means of an AR(1) model.

$$(1 - B^4)GDP_t = 4286 + 0.669(GDP_{t-1} - GDP_{t-5}) + e_t \quad (4-1-7)$$

(1.8) (4.4)

AIC=604, $R^2 = 0.40$ and $\hat{\sigma} = 5553$.

This model is an improvement in terms of AIC and fit and the variability of the residuals. The new residuals are shown in (4-1-7). An alternative formulation of (4-1-7) is

$$GDP_t = GDP_{t-4} + 4286 + 0.669(GDP_{t-1} - GDP_{t-5}) + e_t \quad (4-1-8)$$

We can use the estimated model for forecasts for 2001:4 by

$$GDP_{2001:4} = GDP_{2000:4} + 4286 + 0.669(GDP_{2001:3} - GDP_{2000:3}) + e_{2001:4}$$

Here all terms are known except for the random term $e_{2001:4}$. It is assumed that this term is independent and normally distributed with zero mean. The best prediction for $e_{2001:4}$ is zero. Thus, the forecast for GDP will be

$$GDP_{2001:4} = 532738 + 4286 + 0.669(457783 - 455781) = 538\,363,$$

an increase by 1.1 per cent in comparison with the fourth quarter of 2000.

Graph 4-1-2 shows that the seasonal variation has “disappeared”, but there is still correlation at lag=1.

A further differentiation at Lag=1 is shown below.

$$\begin{aligned} (1 - B)(1 - B^4)GDP_t &= (1 - B)(GDP_t - GDP_{t-4}) \\ &= GDP_t - GDP_{t-4} - (GDP_{t-1} - GDP_{t-5}) \end{aligned}$$

$$\begin{aligned} GDP_t - GDP_{t-4} - (GDP_{t-1} - GDP_{t-5}) &= -339 + 0.492[GDP_{t-1} - GDP_{t-5} - (GDP_{t-2} - GDP_{t-6})] \\ &(-0.5) \quad (7.5) \end{aligned}$$

AIC=555, $R^2 = 0.67$ and $\hat{\sigma} = 3386$, a clear improvement compared to model (4-1-7). The model can be written in a simpler way by using the operator $B^4(y_t) = y_t - y_{t-4}$ as

$$GDP_t = -339 + GNP_{t-4} + 1.492(1 - B^4)GDP_{t-1} - 0.492(1 - B^4)GDP_{t-2} + e_t \quad (4-1-9)$$

GDP is determined by GDP the same quarter last year plus a weighted sum of earlier changes of GDP. The variability of the new residual has decreased but there is still a source of variability not taken care of, i.e. the number of working-days. This dependency and its estimation are not shown here but require more elaborate investigation.

The estimated model (4-1-9) can also be used for forecasting GDP for the 4.th quarter 2001. The forecast is 532 747, i.e. an unchanged GDP in comparison with the 4.th quarter 2000. This forecast has higher probability in terms of likelihood because the model used is better than model (4-1-7), which indicated an increase of 1.1 per cent. Predictions of GDP with model (4-1-9) are shown in graph as well as the forecast for the 4.th quarter 2001²⁹. In model (4-1-9), we have made two differentiations, the first at the seasonal frequency and the other at lag=1 for the non-seasonal part, i.e. we have estimated an ARIMA(110)(010) for GDP. The transformations $d=1, D=1$ have been set in order to get a stationary series. Then we estimated the AR(1) parameter with the SAS software and its value was 0.49. The model was used for a forecast.

4.1.3 Moving average models

In this chapter, we show some other models for GDP starting with a moving average model. We have already seen- and we know that the original series of GDP show seasonal variation with a seasonal period of 4. A possible model would be

$$(1 - B^4)GDP_t = m + \varepsilon_t$$

where m is a parameter and ε_t the residual. An estimated model is

$$(1 - B^4)GDP_t = 13363 + e_t \quad (4-1-10)$$

The residuals are shown in graph 4.1.10 and they appear to be highly correlated, e_t and e_{t-1} have strong positive autocorrelation according to

$$e_t = 0.61e_{t-1} + \varepsilon_t \quad (4-1-11)$$

where ε_t is uncorrelated. (4-1-10) and (4-1-11) can be put together according to

$$(1 - B^4)GDP_t = 12691 + 0.61e_{t-1} + e_t \quad (4-1-12)$$

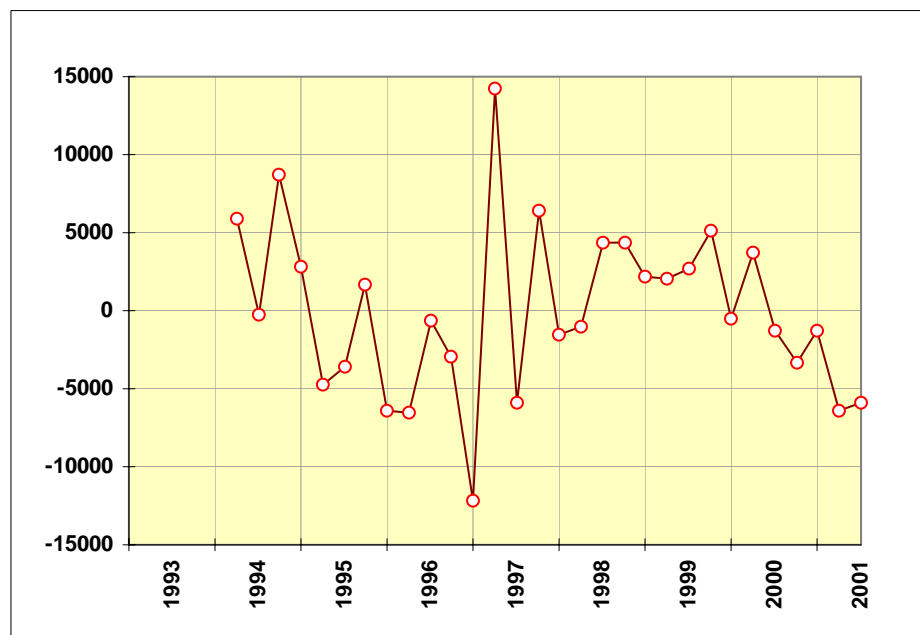
²⁹ By taking differences of a series, we lose observations in the beginning and at the end of a series. These losses of data could be 'solved' by forecasting the series.

(4-1-2) is called a moving average process of first order MA(1). By including more terms e_{t-k} for $k=2, \dots$, we will get a MA-process of higher order. Model (4-1-12) is written ARIMA(001)(010).

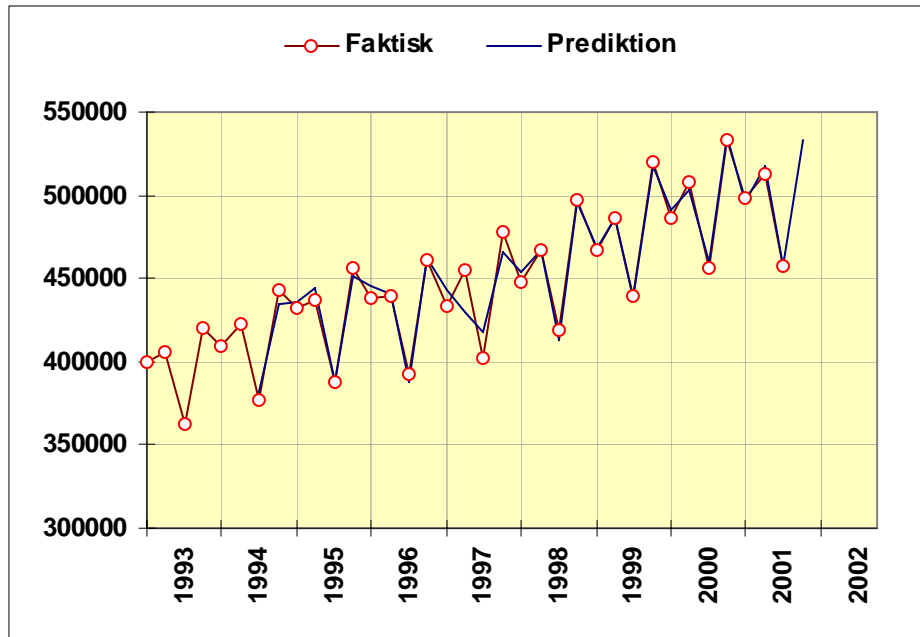
4.1.4 Mixed ARIMA-models

Many time series can be approximated by pure AR-models or pure MA-models. For certain time series, a better description of the data will be achieved by including both AR and MA –terms. There are many statistical tools to use in order to select the ‘best’ model for a particular time series. The BIC-measure introduced earlier is very important and has also been used for the identification of a proper model at Statistics Sweden. The choice of an ARIMA-model for the series is very important and has a crucial impact on the actual seasonal adjusted numbers shown to the user of statistics. This is further discussed in chapter 5.1. However, there are many other important dimensions of seasonal adjustment. The choice of software is also very important. The software TRAMO/SEATS used at Statistics Sweden is briefly described in the next chapter

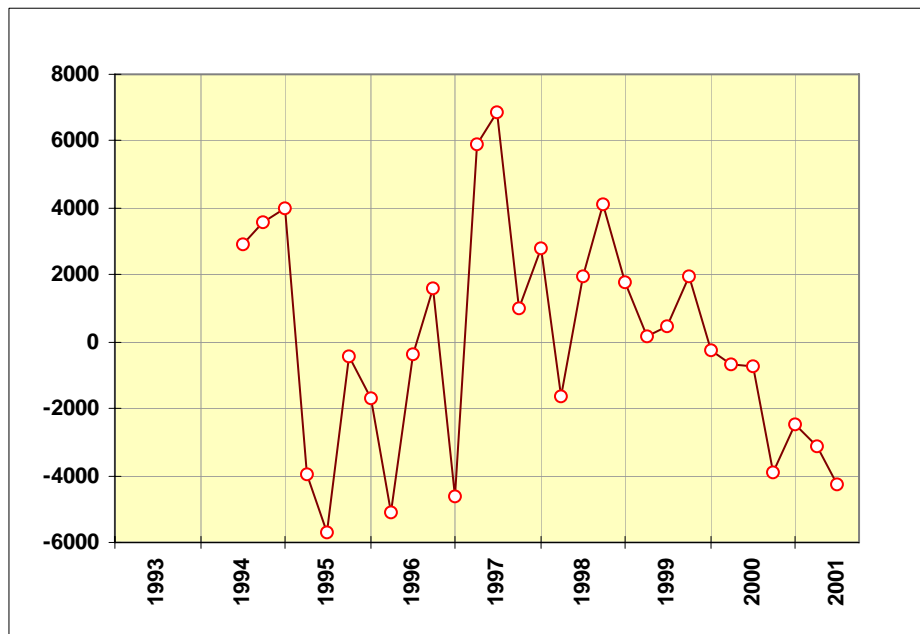
Graph 4.1.2 Residuals from Model (4-1-7) for GDP.



Graph 4.1.3 The Prediction of³⁰ GDP. Model 4-1-9.

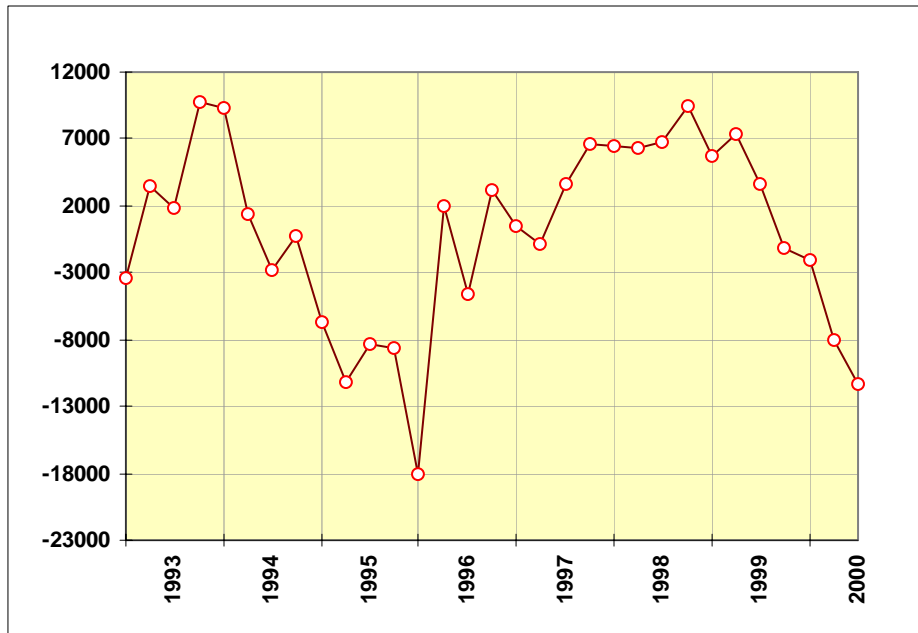


Graph 4.1.4 Residuals from Model 4-1-9.



³⁰ In this graph and the following graph 'Faktisk' stands for 'Actual' in English, 'Prediction' for 'Prediction' in English.

Graph 4.1.5 Residuals from Model (4-1-10).



4.2 TRAMO

This chapter is more technical and some statistical background seems necessary. Seasonal adjustment with TRAMO/SEATS is made in two steps. In the program TRAMO a regression model with an error following a general ARIMA-model. Given the time series $\mathbf{y}_t = (y_1, \dots, y_t)$ of observed values, TRAMO estimates the regression model

$$\mathbf{y}_t = \mathbf{x}'_t \boldsymbol{\beta} + v_t \quad (4-2-1)$$

where \mathbf{x}_t is a vector of m regression variables

$$\mathbf{x}'_t = (x_{1t}, \dots, x_{mt}), \quad (4-2-2)$$

$\boldsymbol{\beta}$ is a vector of m unknown parameters.

$$\boldsymbol{\beta}' = (\beta_1, \dots, \beta_m) \quad (4-2-3)$$

and v_t follows a general ARIMA –process depending on parameters Θ .

$\hat{\boldsymbol{\beta}}, \hat{\Theta}$ are estimates of $\boldsymbol{\beta}, \Theta$. All parameters are simultaneously estimated by maximum likelihood (ML). TRAMO produces also a set of diagnostics in order to check if the ARIMA-model is proper. If no ARIMA –model is specified, TRAMO has an algorithm, which searches for a suitable ARIMA-model. The regression model estimates two types of non-seasonal effects, calendar effects and outliers. The following pre-defines regression variables can be used:

- a) dummy variables for additive outliers (AO), level shift (LS) and temporary changes (TC),
- b) the number of working-days, #³¹(Mondays+Tuesdays+...+Fridays)-#Saturdays-#Sundays,
- c) #Mondays-#Sundays,..., #Saturdays-#Sundays,
- d) the number of days in a quarter/month,
- e) Easter effect.

The length of Easter may be set to achieve the best fit to the series. The regression effect for an additive outlier is specified as follows:

$$y_t = \beta x_t + v_t \quad (4-2-4)$$

where

$$x_t = \begin{cases} 1 & \text{if } t=t_0 \\ 0 & \text{if } t \neq t_0 \end{cases} \quad (4-2-5)$$

TRAMO uses a search algorithm based on a significance test for $\hat{\beta}$ for all time points in the series. It is similar to stepwise regression, in which significant outliers are included one by one. If $\hat{\beta}$ is significant for $t=t_0$ at the chosen significance level, the effect $\hat{\beta}$ at time $t=t_0$ is included. A level shift (LS) is defined according to

$$x_t = \begin{cases} 1 & \text{if } t \geq t_0 \\ 0 & \text{if } t < t_0 \end{cases} \quad (4-2-6)$$

The regression variable for an Easter effect is

$$x_t = \begin{cases} 1 & \text{if } t_0 \leq t \leq t_0 + \ell \\ 0 & \text{otherwise} \end{cases} \quad (4-2-7)$$

where the length of the effect is ℓ is controlled by the user. A regression variable associated with a temporary change is also estimated. Its effect is restricted to a short interval by an exponentially decreasing function. This outlier effect is useful e.g. when a new consumption tax is introduced at $t=t_0$. There is usually a first effect on consumption at $t=t_0$, but the effect will normally decrease at t_0+1 and even more at later times. Sooner or later, the consumers have forgotten and there is no effect³². The effect of the number of working-days has been included in the model. The variable used is defined according to the Swedish calendar.

³¹ # is used for 'the number of'.

³² Examples of these effects are given in table 5.4.1.

$$x_t = \frac{\text{the number of working days during quarter } t}{\text{the average number of working days during a quarter}} \quad (4-2-8)$$

The ratio (4-2-8) is used to calculate an additive working component such that the sum of the components are zero over a year³³. The program estimates the parameters with ML or with least squares further described in Gómez and Maravall (1992,1994), Gómez, Maravall and Pena (1996). The program uses the Kalman filter and other extended filters described in Gómez and Maravall (1993) for the forecasts of the time series used in SEATS. The outliers are treated 'one by one' according to a method as described in Tsay (1984).

The estimated outliers, trading days and working-day effects are eliminated before the decomposition of the time series in the components made in SEATS. The residual series $y_t - \mathbf{x}'_t \hat{\beta}$ (the linearised series) is input to SEATS. The reason to eliminate these effects is that the decomposition of the series into a seasonal, trend, etc. can be made more efficient. The decomposition made in SEATS is briefly described in the next chapter.

4.3 SEATS

The decomposition³⁴ of the linearised series³⁵ $v_t = y_t - \mathbf{x}'_t \hat{\beta}$ is made in the program SEATS (Signal Extraction of Arima Time Series) according to the

$$v_t = \sum_i v_{it} \quad (4-3-1)$$

where the components in SEATS are v_{pt} = the trend, v_{st} = the seasonal, v_{ct} = the cycle and v_{ut} = white noise

The classification of the components are based on the properties in so called the frequency domain, i.e. the spectral density function³⁶ of the components. The trend represents the long-term movements with a spectral peak at frequency zero. The seasonal components have spectral peaks at the seasonal frequencies. The spectrum of the error term should be flat³⁷. Every component in (4-3-1) is given an ARIMA representation³⁸. Imposing restrictions in the

³³ This is important because there should be no overall effect for a whole year, unless there is a leap year.

³⁴ The statistical foundation for this decomposition with SEATS is given in Cleveland and Tiao (1976), Box, Hillmer and Tiao (1978), Burman (1980), Hillmer and Tiao (1982), Bell and Hillmer (1984), Maravall and Pierce (1987).

³⁵ The effects of outliers and calendar have been removed by TRAMO.

³⁶ An example of the of spectral analysis is shown in e.g. Fishman (1969), chapter 3.

³⁷ The notation 'white noise' refers to the spectral decomposition of the frequencies in the light the eye can see. If different 'colours', i.e. frequencies, of the light are mixed it will turn to white.

³⁸ This is consistent with the ARIMA-model for the original series.

model makes the identification of the parameters³⁹. This is called 'the canonical property'. Mathematically, it means that the components are made orthogonal to each other. This is made consistent with the ARIMA-model for their sum v_t . Before the estimation of components is made, the linearised series is forecasted/backcasted two years in order to use symmetric filters⁴⁰. The filters used are of type Wiener-Kolmogorov. If the ARIMA-models used are correct, the filters have been proven to be optimal for linear filters in statistical sense. It means that if the used model is 'proper', the estimation of the components and the reduction of noise is the best that could be made in receiving the 'signals' for seasonal, trend, cycle and noise.⁴¹

5. THE IMPLEMENTATION OF SEASONAL ADJUSTMENT

The arguments in favour for the software TRS have been discussed earlier in this report. That does not mean that we have 'solved' the problem of SA. The software has to be implemented within a production system. There are also many parameters to be set, i.e., choices for SA of a particular series. There are two separate routes to follow. Either we can use the default parameters or we can make such choices individually/manually for every series. SCB has taken the last approach for all national accounts series. That means that every series has been individually analysed in order to maintain good quality for every single series. The implementation of TRS for the national accounts is further discussed in this chapter.

5.1 The Choice of the ARIMA-Model

Model-based SA is founded in time series analyses. A very important moment is to forecast/backcast the original series in order to use symmetric filters. If these forecasts are good, the SA revisions of the series will be small and consistent with the variability and uncertainty of each series. Before a forecast can be made, a model for the original series has to be chosen and estimated. In TRS the ARIMA-models are used in a standard way. The specification of the ARIMA-model includes several moments in line with the Box-Jenkins approach. This can be done in many ways. The most important principle for the choice of the model is the likelihood-principle. The model is chosen, which gives the best explanation of the actual outcome, i.e. given the data. The BIC-measure is used, which gives a penalty for overparametrization of the model. These issues have been discussed in chapter 4.1.1. The relevance of the BIC-measure lies on the assumption that the error distribution is normal and independent. For that reason, it is very important to continuously check that the underlying assumptions are met. This is done routinely in TRS by statistical

³⁹ See Maravall (1995), page 32.

⁴⁰ Similar approach is also made in X11-ARIMA and X-12-ARIMA.

⁴¹ The methods used in X-12-ARIMA do not share this property. The filter used by SEATS depends on the empirical properties of the series and the model. They are adaptive to what is empirically verifiable. X-12-ARIMA uses fixed filters which are quite good for certain series, but quite inefficient filters for other series. A further discussion of filter design, see e.g. Monson H. Hayes (1996), *Statistical Digital Signal Processing and Modeling*. JWS.

tests⁴². A number of parameters have to be estimated. Good quality in SA assumes that the model is significantly estimated and stable over time⁴³. For all series SA in NA, 50 different ARIMA-models have been investigated. The ultimate choice has been made from these models according to the following criteria:

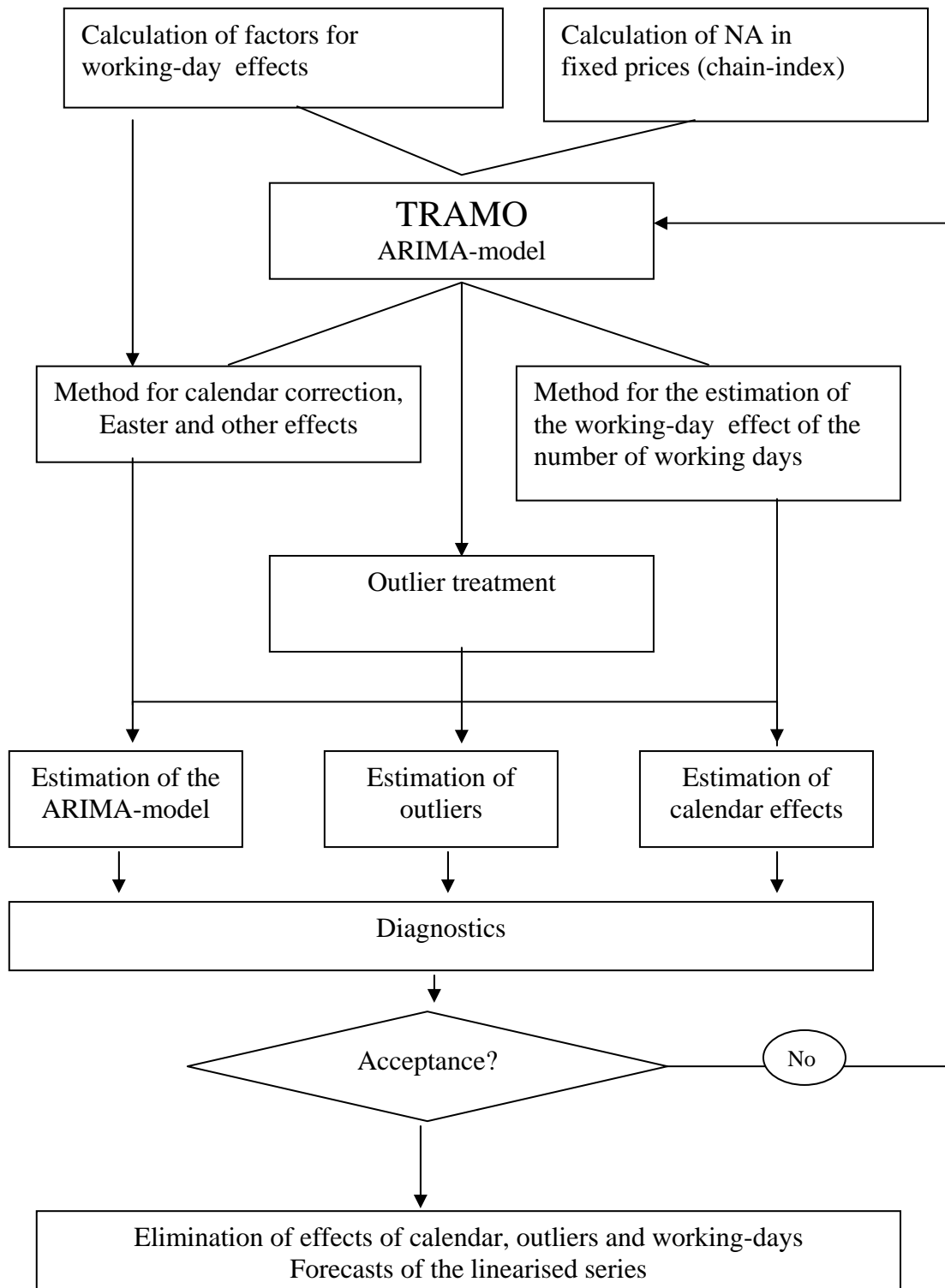
- K1 Maximum likelihood (BIC),
- K2 Statistical tests of the residuals
- K3 Autocorrelations of the residuals
- K4 Graphs of the residuals
- K5 Significance of the parameter estimates of the ARIMA-model
- K6 Variability of the SA series
- K7 Graphs of the SA series

The different moment of NA at SCB is described in graph 5.1.1.

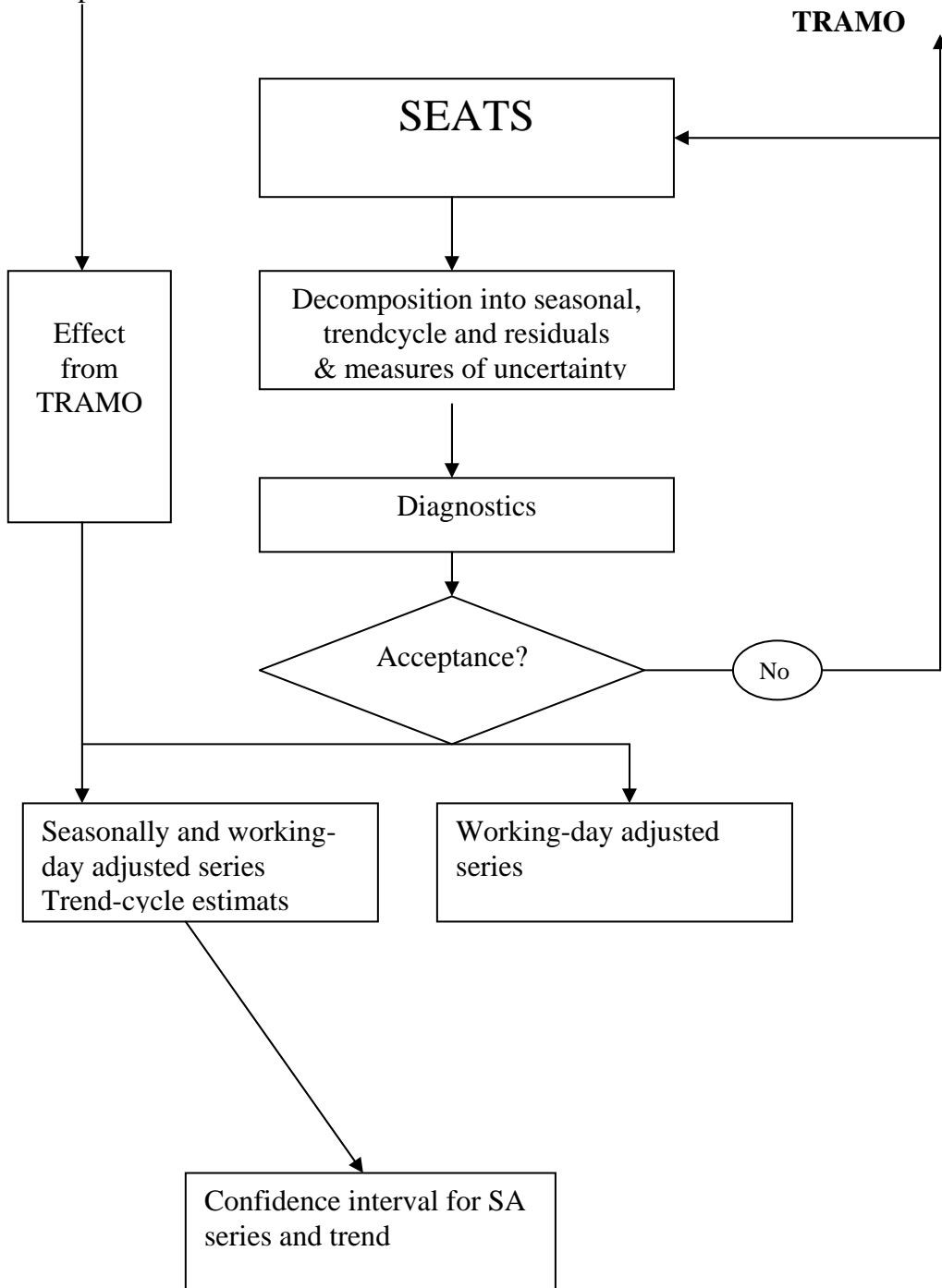
⁴² Test of normal distribution, Durbin Watson test of autocorrelation and Q-test of autocorrelation of the residuals. See Ljung and Box(1978) and Pierce(1978).

⁴³ The parameters of the model for the calendar/workin-day effects included.

Graph 5.1.1 Seasonal Adjustment of National Accounts



Graph 5.1.1



In those cases a unique best model according to the criteria K1,K2 and K5 exists, it has been chosen. This is very unusual. Normally, the distribution of BIC among models does not discriminate between models enough for a unique choice of the best model. Among the models showing approximately the same likelihood according to BIC, a further screening take place based on the properties of the residuals and the tests available of the programs. Any significant diagnostic test rejects the model. Graphs of the residuals and also of the SA series and the trend estimates and the changes are used as complementary information eventually showing more complex patterns of the residuals, not revealed by the tests. Models with not statistically significant parameter estimates are normally rejected⁴⁴. If two or more models have the same quality rank according to the used criteria, the model showing the lowest variance of the changes of the SA series is chosen⁴⁵.

In order to illustrate some of the analyses made for the choice of an ARIMA-model for GDP, we show the distribution of the BIC-measure for 50 ARIMA-models in table 5.1.1 below.

Table 5.1.1 Distribution of BIC for 50 ARIMA-models of GDP.

BIC	Frequency	Per cent	Cumulative frequency	Cumulative frequency Per cent
17.1	4	8	4	8
17.2	3	6	7	14
17.3	7	14	14	28
17.4	11	22	25	50
17.5	8	16	33	66
17.6	2	4	35	70
17.7	1	2	36	72
17.8	4	8	40	80
17.9	1	2	41	82
19.5	1	2	42	84
19.8	1	2	43	86
20.2	1	2	44	88
20.3	1	2	45	90
20.4	1	2	46	92
20.8	1	2	47	94
20.9	1	2	48	96
21	1	2	49	98
21.1	1	2	50	100

⁴⁴ Unless there is no acceptable model with significant parameters. The best model in that 'inferior' class is chosen.

⁴⁵ This consideration of 'low' variability is of great concern for SCB because SCB has started to publish monthly/quarterly changes of SA values raised to a yearly level. Because the noise of the SA series will also be raised to a yearly level, the signal to noise-ratio must be kept at an 'acceptable' level. For instance, a change of GDP of say 15 % at an yearly level is not acceptable.

The best model in the sense of BIC has BIC=17.1. The four best models are ARIMA(003)(010), ARIMA(012)(010), ARIMA(013)(010), ARIMA(013)(011) with BIC=17.1. These models are equally likely in terms of BIC. The most unsatisfactory model in the sense of BIC, has a value of 21.1. There are four models with BIC=17.1 shown in table 5.1.2 below. They can be said to be statistically equivalent. These models are called model 17, 19, 20 and 39 (among the 50 models).

The ARIMA-parameters and the standard errors (SE) of estimates are shown in model 17, there are three MA parameters 0.28 0.78 and 0.62. * means that there is no estimate made⁴⁶. For all these models there are at least some problems in terms of the statistical significance of the parameters and/or numerical problems as indicated. We consider they all as uncertain models.

Table 5.1.2 Estimated ARIMA-models for GDP

Model	ARIMA	Parameter-estimates
17	(003)(010)	0.2776 0.7820 0.6185 SE=*,*, 0.1341
19	(012)(010)	-0.5106 0.9800 SE=*,*
20	(013)(010)	-0.3369 0.8571 0.2304 SE=*,*,0.1752
39	(013)(011)	-0.3177 0.8906 0.1911 SE=*,*,0.1859 <u>Seasonal part</u> 0.2763 SE=0.1771

All models of table 5.1.2 all show acceptable residual properties. We show a graph of model 17 in graph 5.1.1 on page 32.

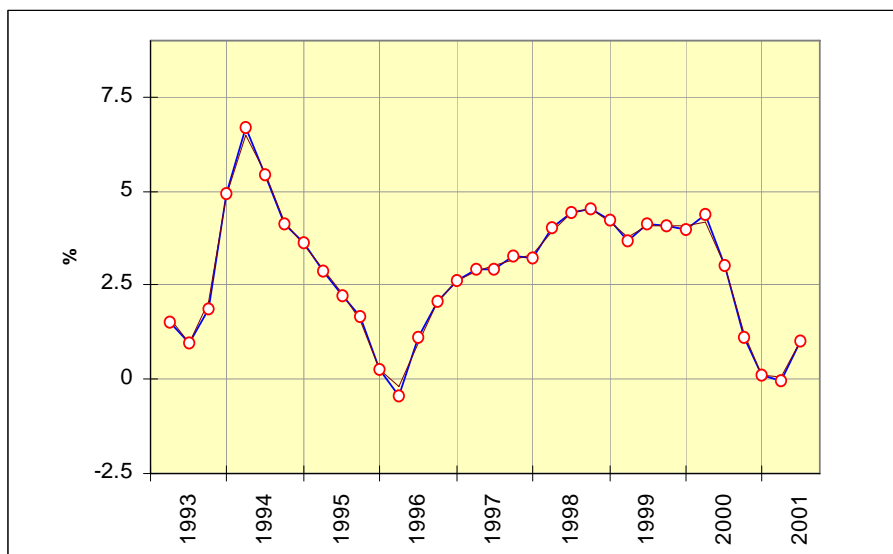
The seasonally adjusted and working-day-adjusted series is shown with a small circle in the graph. The trend estimate is shown as the thin line. As can be seen there is very hard to separate the SA series and the trend. We consider model 17 as very odd, because the filter for the SA series and the trend estimate are almost equal.⁴⁷

⁴⁶ If no estimate of the standard error is produced, we can not make any statistical inference about the true parameter of the model. The reason may be numeric problems or other problems with the model. In any case, we consider the model 'not reliable'.

⁴⁷ Some users of statistics in Sweden have noticed this 'problem'. They claim that the SA series and the trend estimates are almost identical even in the case of the finally chosen ARIMA-model (110)(010) producing the changes of SA series as shown on the first page. Very informed user of statistics in Sweden would accept model 17 and the graph 5.1.1.

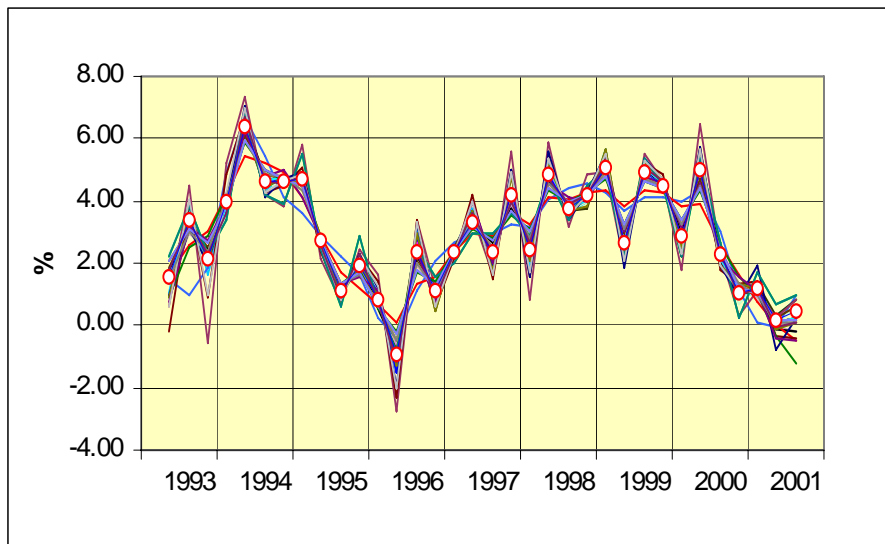
The last point concerns the parametric aspect of the estimated possible ARIMA-models and their BIC-measure. Other considerations are discussed later. For all national accounts series 50 ARIMA-models have been analysed. The final choice of an ARIMA(110)(010) was made in 1999, i.e. an AR(1) with a significant parameter⁴⁸ of 0.37 and BIC=17.3, almost as good as the models shown in table 5.1.2. It has passed all diagnostic tests since 1999 and proven to be quite stable. It also produces quite clear signals of peaks and downturns of the business cycle. The model and all other models is under observation and surveillance. It's significance properties have sometimes been on the border of questioning in favour of a competing model. On this matter, SCB is conservative. There must be very clear indication for more than once that a model should be replaced by some new model. A new model show a new picture of the economy in terms of SA numbers not only for the last observations but also for the whole series as will be discussed in the next section.

SA and working-day corrected GDP and trend.
Quarterly changes raised to a yearly level. Per cent
ARIMA(003)(010).



In the first place we can observe that the choice of the ARIMA-model is important for all times⁴⁹ not only at the end of the series. The range is about two per cent. Even if we eliminate the most extreme models, there is a remaining ‘model variability’ of about 1 per cent. If we look at the third quarter of 2001, we have a mean change of about 0.4 per cent with a standard deviation of 0.7. The change from the official estimates was 0.2 per cent, i.e. in the middle of the distribution of changes⁵⁰. For most of the NA series, there has been possible to identify a proper ARIMA-model, which focus of comparisons of the SA series over time. Because of the quite short period, there is a considerable need for statistical surveillance of the specifications including the choice of the ARIMA-models.

SA and working-day adjusted GDP
Quarterly changes in per cent raised to a yearly level
50 ARIMA-models



Graph 5.1.2

5. 2 Diagnostic Tests

The identification of the ARIMA-models for NA series has been described in chapter 5.1. The production system for regular adjustments is designed for instant attention to SA difficulties through the diagnostics. For every series, the SAS-interface produces:

- D1. Estimated parameters of the ARIMA-model and the standard errors.
- D2. Statistical tests as discussed in chapter 5.1. Colours are used to call attention to potential problems and deviations from the SA assumptions (red colour). Blue is used to signal ‘no problem’ and green indicates ‘perfect’.

⁴⁹ The reason for this is that TRS uses the estimated parameters in the filter used in the decomposition of the series.

⁵⁰ That is also the case for other quarters.

- D3 Residuals
- D4 Autocorrelation of the residuals
- D5 Graphs of the residuals,
- D6 Outlier effects, value and date
- D7 Working-day effects,

136 official time series were seasonally adjusted in 1999 in regular production. There were 272 infiles of the system, 580 files of output, a lot of things to be kept in surveillance. However, the IT-system is designed for easy maintenance and control running on a server at NA. All series are processed at the same time in batch-mode.

5. 3 Working-day and Calendar Effects

Calendar effects are estimated in TRAMO using the regression model (4.2.1). The working day variable ‘the number of working-days in a quarter’ is used as a ratio (as compared to a yearly average) for all series of value added and also employment series. The working-day variable according to (4-2-8) is calculated at the lowest sector level and aggregated to higher levels by means of value added. Generally, the ratio-method used by SCB for working-day adjustment, produced a too large working-day effect. For GDP approximately 40 per cent of the gross effect (as measured by the input to TRAMO) is empirically verified. This effect is statistically significant for GDP and total employment but in many cases not significant at lower aggregation levels. Generally, the working-day effect earlier used by SCB cannot be verified empirically. For an example, the empirically verified working-day effect for GDP is just about 40 per cent of the effect earlier used as calculated directly from the calendar⁵¹. The estimated working-day effects for GDP and for total number of hours worked are statistically significant. The calendar effects are not always statistically significant on low aggregation levels. Comparisons between not adjusted⁵² working-day effects and estimated by a regression model are shown in table 5.3.1 for GDP and other important NA series and in graph 5.3.1-5.3.4. The bias of the ratio-method could be verified from the table and from the graphs. In some sectors, the ratio method and the regression method give the same working-day effects (SNI=10-14). For many series, the working-day effects are insignificant.

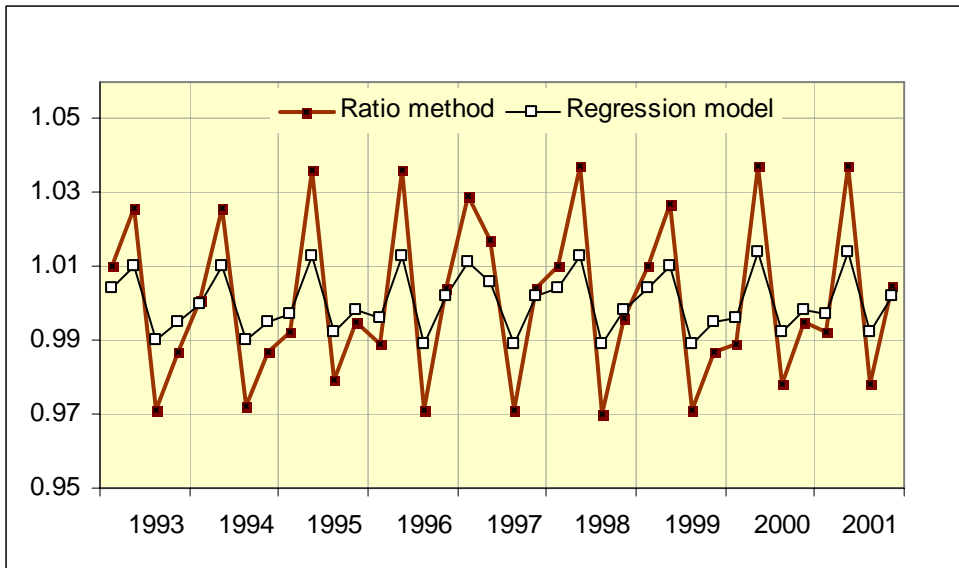
A summary of the working-day effects would be as follows. Before 1999, the national accounts used a simple ratio-adjustment based on the number of working-days for all value-added series. This method was biased in the sense that the working-day adjusted series were ‘overcompensated’ for the working-day effects. As a consequence of the bias, the ratio method also produced ‘extra’ variation of the SA series and their changes. Since 1999, SCB has used a regression method, which estimates smaller working-day effects with lower variability.

⁵¹ These are calculated by simple ratios before SA

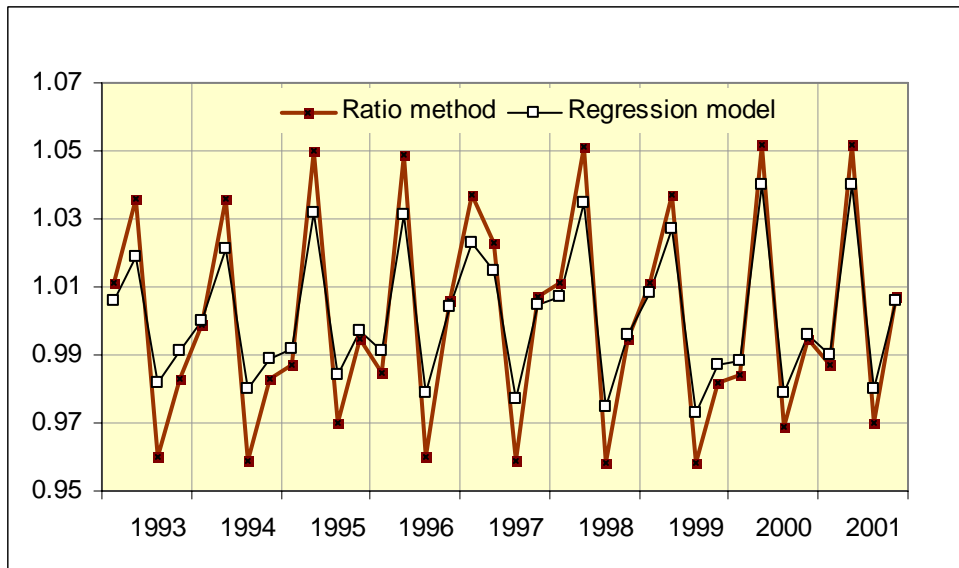
⁵² These are calculated by simple ratios before SA.

Table 5.3.1 Unadjusted and Estimated Calendar Effects in National Accounts

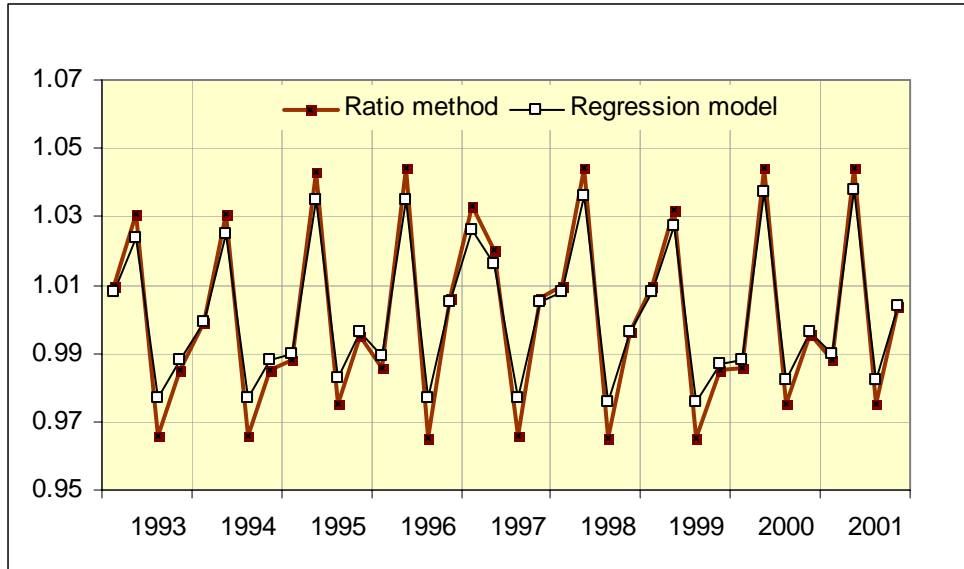
	GDP		Manu- facture		Number of hours worked		Number of hours worked Manufacture	
	Unad- justed	Estima- ted	Unadjusted	Estimated	Unadjusted	Estima- ted	Unadjusted	Estimated
1993:q1	1.010	1.004	1.011	1.006	1.010	1.008	1.010	1.006
:q2	1.026	1.010	1.036	1.019	1.031	1.024	1.037	1.022
:q3	0.971	0.990	0.960	0.982	0.966	0.977	0.958	0.980
:q4	0.987	0.995	0.983	0.991	0.985	0.988	0.983	0.990
1994:q1	1.001	1.000	0.999	1.000	0.999	0.999	0.997	0.998
:q2	1.026	1.010	1.036	1.021	1.031	1.025	1.037	1.022
:q3	0.972	0.990	0.959	0.980	0.966	0.977	0.958	0.980
:q4	0.987	0.995	0.983	0.989	0.985	0.988	0.982	0.990
1995:q1	0.992	0.997	0.987	0.992	0.988	0.990	0.984	0.990
:q2	1.036	1.013	1.050	1.032	1.043	1.035	1.052	1.032
:q3	0.979	0.992	0.970	0.984	0.975	0.983	0.969	0.984
:q4	0.995	0.998	0.995	0.997	0.995	0.996	0.995	0.997
1996:q1	0.989	0.996	0.985	0.991	0.986	0.989	0.983	0.989
:q2	1.036	1.013	1.049	1.031	1.044	1.035	1.053	1.032
:q3	0.971	0.989	0.960	0.979	0.965	0.977	0.957	0.979
:q4	1.004	1.002	1.006	1.004	1.006	1.005	1.009	1.005
1997:q1	1.029	1.011	1.037	1.023	1.033	1.026	1.038	1.023
:q2	1.017	1.006	1.023	1.015	1.020	1.016	1.024	1.014
:q3	0.971	0.989	0.959	0.977	0.966	0.977	0.957	0.979
:q4	1.004	1.002	1.007	1.005	1.006	1.005	1.009	1.005
1998:q1	1.010	1.004	1.011	1.007	1.010	1.008	1.010	1.006
:q2	1.037	1.013	1.051	1.035	1.044	1.036	1.053	1.033
:q3	0.970	0.989	0.958	0.975	0.965	0.976	0.957	0.978
:q4	0.996	0.998	0.995	0.996	0.996	0.996	0.995	0.997
1999:q1	1.010	1.004	1.011	1.008	1.010	1.008	1.010	1.006
:q2	1.027	1.010	1.037	1.027	1.032	1.027	1.038	1.024
:q3	0.971	0.989	0.958	0.973	0.965	0.976	0.957	0.978
:q4	0.987	0.995	0.982	0.987	0.985	0.987	0.982	0.989
2000:q1	0.989	0.996	0.984	0.988	0.986	0.988	0.983	0.989
:q2	1.037	1.014	1.052	1.040	1.044	1.037	1.053	1.034
:q3	0.978	0.992	0.969	0.979	0.975	0.982	0.969	0.984
:q4	0.995	0.998	0.995	0.996	0.9954	0.996	0.995	0.997
2001:q1	0.992	0.997	0.987	0.990	0.988	0.990	0.983	0.990
:q2	1.037	1.014	1.052	1.040	1.044	1.038	1.053	1.033
:q3	0.978	0.992	0.970	0.980	0.975	0.982	0.969	0.985
:q4	1.005	1.002	1.007	1.006	1.004	1.004	1.000	1.000



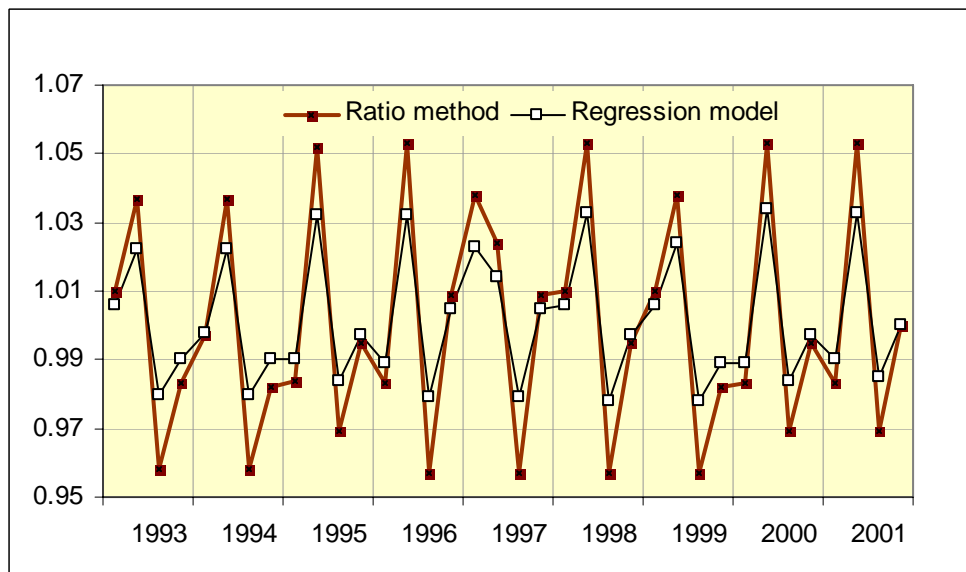
Graph 5.3.1 Working-day Effects for GDP



Graph. 5.3.2 Working-day Effects for Value Added in Manufacture.



Graph 5.3.3 Working-day Effects for the Total Number of Hours worked



Graph. 5.3.4 Working-day Effects for the Number of Hours worked in Manufacture

5. 4 Outliers

Different types of outliers are estimated in TRAMO/SEATS according to chapter 4.2. The knowledge of outliers, type, magnitude and date, could be used for different purposes. It could be used in SA in order to give a more certain estimate of the seasonal component. It could also be used as a tool for quality management of national accounts and the data-sources⁵³. Information on outliers can also be of great value to the user of statistics. If there are outliers in the series, comparisons over time should be made with consideration. See e.g. the change of local consumption between the fourth quarter of 1997 and the first quarter of 1998 in the graphs shown Appendix 1.

There is no outliers in GDP and in total number of hours worked. The outliers estimated in the most important series of NA are shown in table⁵⁴ 5.4.1. The series government consumption shows an additive outlier the fourth quarter of 1996, about 6 per cent higher value than 'normal'. This outlier effects only this quarter and the series are back to normal the first quarter 1997. There is a temporary change between the third quarters of 1999 to the second quarter of 2000. This temporary change starts with an instant change of 6 per cent increase, which is reduced to an increase of 4 per cent the next quarter and so on. The strong impact of these outliers is seen by the corresponding graphs in Appendix 1, especially the quarterly changes. The outlier effects for multiplicative models are shown as index numbers in table 5.4.1. Series with additive models show outliers in value-terms (million Skr). For local government consumption, there is a level-shift of 4674 the first quarter of 1998 and for investments, a temporary change the third quarter of 1997. There are also outliers in import of services.

What about the causes of outliers? In some cases, we have information on the causes behind outliers. There may be changes in the population, changes in definitions. In other cases, outliers may be caused by errors in statistics. When SCB has explicit knowledge on the causes of outliers; they are published as meta-data in tables and in statistical databases.

⁵³ The application of SA for quality control has been done for short-term industrial indicators (Öhlén, 2000:2). The analyses of outliers from a quality point of view are made on a regular bases at NA since 2001.

⁵⁴ The table is based on NA series covering 1993-2001.

**Table 5.4.1 Outliers in Swedish National Accounts.
 Main Components.**

	Household Consumption	Government Consumption	Local Government Consumption	Exports of services	Imports of goods and services
1994:q1					
:q2					
:q3					
:q4					106
1995:q1					104
:q2					103
:q3					102
:q4					101
1996:q1					101
:q2					101
:q3					101
:q4		105			100
1997:q1					
:q2	102	102			
:q3					
:q4					
1998:q1			5641		
:q2			5641		
:q3			5641		
:q4			5641		
1999:q1		107	5641		
:q2		105	5641		
:q3		104	5641		
:q4	103	102	5641		
2000:q1	103	102	5641		
:q2	103	101	5641		
:q3	103	101	5641		
:q4	103	101	5641		
2001:q1	103	103	5641		
:q2	103	103	5641		
:q3	103	103	5641		
:q4	103	103	5641		
2002:q1	103	103	5641	88	
:q2	103	103	5641		
:q3	103	103	5641		
:q4	103	103	5641		
2003:q1	103	103	5641		
:q2	103	103	5641		
:q3	103	103	5641		

5. 6 The Revisions of Specifications

The Swedish national accounts cover the period 1993-2003, the results shown in this report, from 1993 to 2001. From the point of view of SA and model building, this is a very short period. The different sources of variation discussed in chapter 2 may have large effects on the choice of the 'best' model, the need for revision of a model and the specifications of the parameters of the SA procedures⁵⁵. The inclusion of a new observation may have large effects on the seasonally adjusted series especially at the end of the series. Once a year, there are 'large' revisions of the original data on NA covering the last two years. This is done when the third quarter of a year is published, in December. When such revisions are made, there is also a starting point for revisions of SA of all the NA series. If there is any sign of changes of the used options including the used ARIMA-model, a new identification of an ARIMA-model is carried through in lines with chapter 5.1. Approximately 20 % of the specifications of 1999 have been changed to maintain high quality of SA and facilitate proper comparisons over time.

5. 7 Measures of Uncertainty

The software TRAMO/SEATS used at SCB estimates measures of uncertainty for the estimated components of the model (2-1) or (2-3)⁵⁶. These measures of uncertainty are shown separately for every time series in the production system at SCB. Based on the standard errors of the estimated components, it is possible to calculate a confidence interval for e.g. the SA value. Such 95 % confidence intervals are shown in the graphs of Appendix 1. The point estimate of the SA value the third quarter of 2003 is 574748 and the 95 per cent confidence interval is 571938—577557. The length of this interval is about .5 per cent of the SA value⁵⁷. We should be quite certain that 'true' SA GDP would be in that interval. However, it should be kept in mind that a negative change of GDP between the second and third quarter 2003 is consistent with the confidence interval⁵⁸ although not very likely. We should be pretty certain that seasonally adjusted GDP would be in this interval. However, we should be aware of the possibility that the 'true' value for the third quarter 2001 admits a positive change but also a negative change. The length of the confidence interval for household consumption expenditures is 0.4 per cent.

5. 8 Time Consistency

As discussed earlier, time consistency, i.e. yearly totals and yearly totals of SA series are equal, should be considered. The earlier research at SCB has indicated that time consistency is not a serious problem. There seems to be arguments among experts on time series analyses and SA against this requirement. Any adjustment of the SA series to achieve time consistency would reduce the quality of seasonal adjustment. If the SA procedure is

⁵⁵ E.g. the choice between an additive/multiplicative models.

⁵⁶ The standard error of the estimated component.

⁵⁷ $100 \cdot (571938 - 577557) / 574748$.

⁵⁸ The left limit of the interval, 571938, is less than the SA value for the second quarter, 574747.

optimal in statistical sense, a time consistency restriction would change the optimal properties of the procedure. However, many users like the idea of time consistency. When time consistency can be achieved within the SA procedure, it has been made. In the presence of deterministic effects as outliers, time consistency cannot and has not been achieved within the seasonal adjustment of the series. However, there are quite small differences between yearly totals of original series and the SA series. This can be seen from table 5.8.1 for the components of GDP. The table shows yearly differences in per cent as well as the mean, standard deviations and min/max-values. The largest deviation for GDP is -0.31 per cent of the original series for 1993-2002. The largest deviations can be seen for the exports and imports, about 0.5 per cent in 1998. Time consistency is approximately achieved for all components of GDP but not on all aggregation levels for value added.

Table 5.8.1 Yearly Deviations between seasonally Series and original Series in per cent.

	GDP	Household Consumption	General government consumption	Government Consumption	Local Government Consumption	Gross capital formation	Exports of goods and services	Exports of goods
1993	-0.21	-0.01	0.00	0.00	-0.21	0.00	0.36	0.35
1994	-0.31	-0.01	0.00	0.00	-0.38	0.00	0.07	0.11
1995	-0.16	-0.01	0.00	0.00	-0.16	0.00	-0.24	-0.19
1996	-0.12	-0.01	0.00	0.00	-0.04	0.00	0.25	0.20
1997	0.02	-0.01	0.00	0.00	0.07	0.00	-0.39	-0.31
1998	0.07	-0.01	0.00	0.00	0.15	0.00	-0.47	-0.49
1999	-0.16	-0.03	0.00	0.00	0.07	0.00	0.36	0.35
2000	-0.06	-0.01	0.00	0.00	0.09	0.00	-0.09	0.00
2001	0.19	-0.01	0.00	0.00	0.01	0.00	0.15	0.26
2002	0.12	-0.01	0.00	0.00	-0.09	0.00	0.02	-0.11
Mean	-0.06	-0.01	0.00	0.00	-0.05	0.00	0.00	0.02
St. dev.	0.16	0.01	0.00	0.00	0.16	0.00	0.30	0.29
Max	0.19	-0.01	0.00	0.00	0.15	0.00	0.36	0.35
Min	-0.31	-0.03	0.00	0.00	-0.38	0.00	-0.47	-0.49

	Exports of services	Imports of goods and services	Imports of goods	Imports of services
1993	0.00	-0.01	0.05	-0.02
1994	0.00	-0.08	-0.30	-0.03
1995	0.00	0.00	-0.26	-0.02
1996	0.00	-0.01	0.37	-0.02
1997	0.00	-0.04	0.02	-0.05
1998	0.00	-0.01	0.05	-0.01
1999	0.00	-0.02	0.04	-0.03
2000	0.00	-0.01	-0.22	-0.03
2001	0.00	-0.03	0.04	-0.03
2002	0.00	-0.02	0.03	-0.02
Mean	0.00	-0.02	-0.02	-0.03
Stand. dev.	0.00	0.02	0.20	0.01
Max	0.00	0.00	0.37	-0.01
Min	0.00	-0.08	-0.30	-0.05

5.9 Tests with X-12-ARIMA

Eurostat recommends TRAMO/SEATS, but X-12-ARIMA can also be used as a 'second best choice'. The diagnostics⁵⁹ of X-12-ARIMA could be of great value. When SA of the national accounts was implemented in 1999 X-12-ARIMA was also used as a tool for SA.⁶⁰

5.10 Direct or indirect seasonal Adjustment

Direct and indirect seasonal adjustment has been discussed shortly in chapter 3. In this paragraph, this issue will be further discussed. With indirect seasonal adjustment (ISA) we mean that the SA of a time series will be given by aggregation of other SA series. If exports of goods and exports of services are SA, the sum of the SA series will be called exports of goods and services, indirectly SA. The properties of this aggregated series will depend on the properties of the directly adjusted series exports of goods and exports of services. SA of GDP can also be made with the ISA-approach or with a direct approach (DSA). If the components of GDP, household consumption expenditure, government consumption, etc., are SA, the sum of SA components is GDP indirectly SA. Seasonally adjustment with the ISA- and DSA-approach will usually give different results. This is not satisfactory especially if the discrepancies are large.

The following aspects regarding direct or indirect seasonal adjustment have been considered:

- Recommendations made by Eurostat in 1998.
- Theoretical considerations
- Quality aspects

These are shortly discussed below.

5.10.1 Recommendations from Eurostat and ECB

Direct and indirect seasonal adjustment has been discussed at the meetings held by working group for SA at Eurostat and also at the international conference SAM98. At that time, Eurostat recommended DSA on vague arguments. It was pointed out clearly that there was need for further research. ISA for national accounts was an option because of the importance of consistency in national accounts.

⁵⁹ For instance, spectral analysis of the residuals.

⁶⁰ X-12-ARIMA is not used for the moment.

5.10.2 Theoretical Considerations

Suppose the imports of goods and the imports of services follow the same ARIMA-model, it could easily be shown that the series imports of goods and services follows the same ARIMA-model⁶¹. In other terms, if the components have similar dynamic properties, e.g. similar seasonal pattern, ‘similar’ calendar effects and ‘similar’ development over time, we could use indirect seasonal adjustment. If the series do not follow the same ARIMA-model, we cannot be very precise and the differences between the ISA and the DSA-approach will depend on the differences between the ARIMA-models, i.e. the dynamics of the series. The theoretical aspects regarding ISA and DSA are a topic for further research.

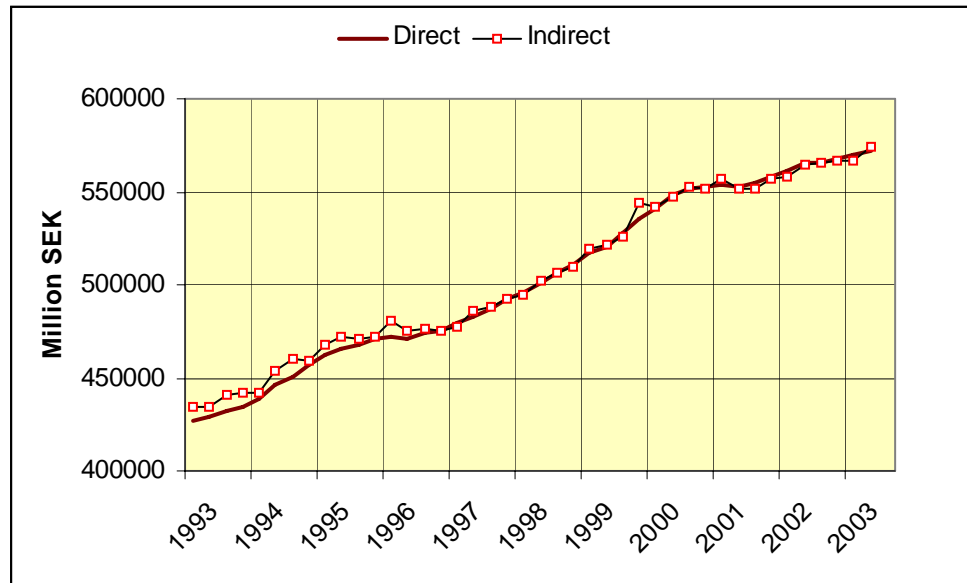
5.10.3 Quality Aspects

The data used for the calculation of the national accounts includes sample surveys, e.g. industrial production. Consequently the quality of the data in terms of e.g. M.S.E. is larger on a sector-level than on an aggregate level. Such differences in quality of the data will also be found in the corresponding time series of the national accounts. For instance, the confidence intervals for the SA series are typically larger at low sector-levels in comparison with the corresponding at higher aggregation levels as indicated by graph 5.11.1 below. DSA at low levels show higher uncertainty at that level because of a corresponding uncertainty of the data. On the other hand, DSA at these levels can be used as a tool for quality control of the data in lines with the discussion of outliers. In the presence of outliers, different working-day/calendar effects, multiplicative/additive models, etc at low levels, the consequences of an aggregation of such effects on higher sector-levels would be unpredictable. The quality of the seasonal adjustment at aggregated levels would also be unpredictable. On the basis of the discussion above, SCB uses DSA for SA of the national accounts. We end this paragraph by showing SA GDP using ISA and DSA and the differences in graph. 5.10.1-3. The largest differences between DSA and ISA, e.g. the quarterly change from the second quarter to the third quarter of 1999 are caused by the presence of outliers among the components of GDP.

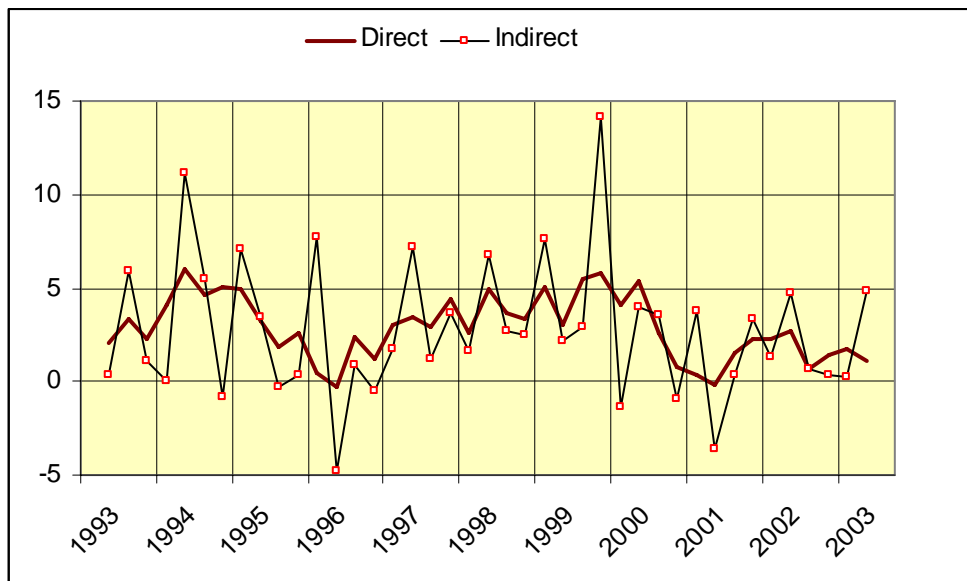
⁶¹ If $x_t \approx \Psi_1(B)a_{1t}$ and $y_t \approx \Psi_2(B)a_{2t}$, then

$x_t + y_t \approx \Psi_1(B)a_{1t} + \Psi_2(B)a_{2t}$. Now, if $\Psi_1(B) = \Psi_2(B) = \Psi(B)$, the sum of the two series will be $x_t + y_t \approx \Psi(B)a_{1t} + \Psi(B)a_{2t} = \Psi(B)(a_{1t} + a_{2t}) = \Psi(B)a_t$,

where $a_t = a_{1t} + a_{2t}$.



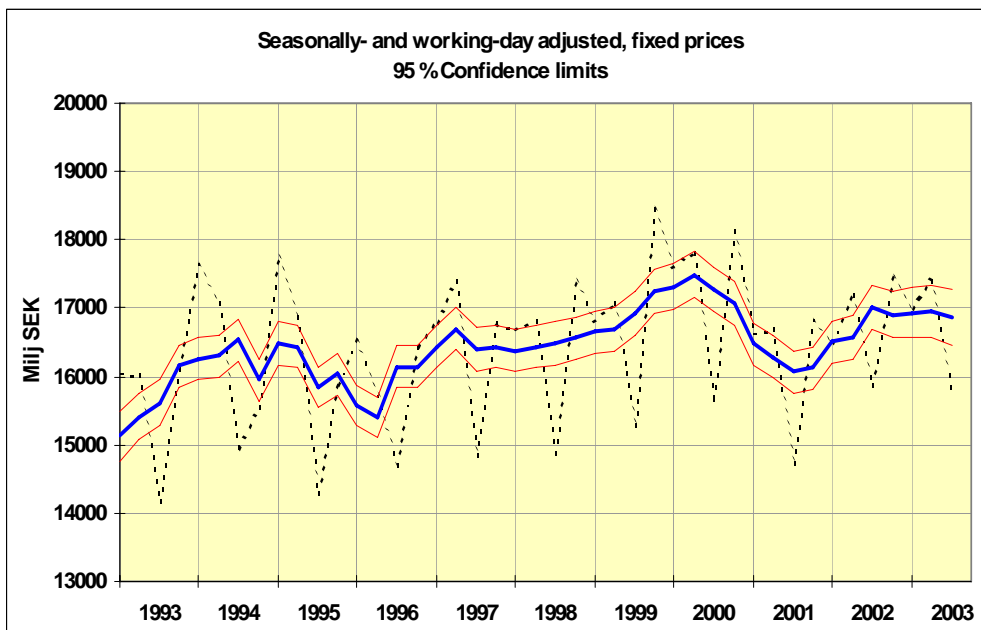
Graph 5.10.1 Seasonally adjusted GDP, Direct and Indirect approaches.



Graph 5.10.2. Seasonally adjusted GDP. Direct and indirect approach. Changes in per cent from preceding quarter raised to a yearly level.

5. 11 The Variability of seasonally adjusted Series

Desirable properties of methods of seasonal adjustment were discussed in chapter 3.1. It was pointed out that a SA series with low variability were preferred before a SA series showing high variability, otherwise of equal quality. SCB chooses the model with low variability within the class of statistically admissible models.



Graph 5.11.1 Production of pulp, paper, printing and publishing.

This smoothness-criterion might seem to be dangerous in the sense that we might remove information in the data in getting ‘too smooth’ SA series. However, this is not the case. All national accounts series with the exception of GDP and total number of hours worked show large variability. The variability concept is further discussed in this chapter, first the principle of low variability and what should be meant by ‘smooth’.

The property of low variability and smoothness of seasonally adjusted series was discussed by Erik Ruist⁶² already around 1965 in an article. He argued that if first differences of the SA series has low variability, i.e. are smooth, comparisons over time would be easy to make, especially for economic data. He suggested a non-parametric algorithm, which could be used for SA of a quarterly time series. This algorithm has been generalised to monthly data with a regression model for working-day and/or calendar effects in Öhlén⁶³. This non-parametric SA method has been compared with TRAMO/SEATS for Swedish national accounts series. TRAMO/SEATS produce SA series with lower variability than this method and also in comparison with X-12-ARIMA.

⁶² Professor in economic statistics at Stockholm School of Economics.

⁶³ Se Öhlén (2000:1).

In order to understand the reasons why TRAMO/SEATS produces SA series with very low variability, it would be necessary to refer to the theory of linear filtering within the framework of Wiener and Kolmogorov. Filter theory and signal extraction has been used since many years in order to reduce noise of different kinds and in different applications. It is used in transmission of electrical and optical signals for communications on the Internet, in cars, in music and in medical diagnostics every second. It is also used by the program SEATS to increase the signal/noise-ratio⁶⁴

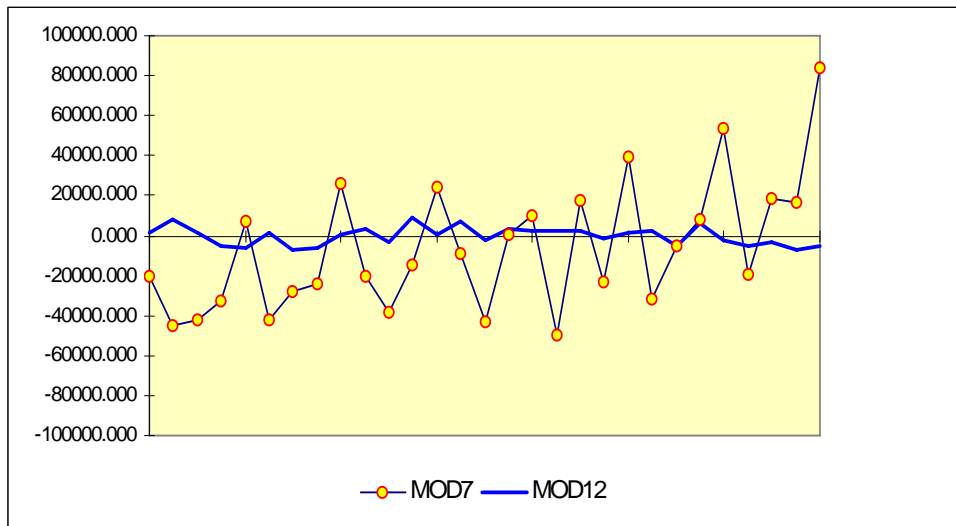
The observed value O_t according to the model (2-1) is the sum of the systematic components and the noise I_t . The process behind the model, the economic system, causes this noise but it is also depending on our ability to explain the systematic part of the variation over time in terms of the components of the model. A bad choice of a model will induce a large error-term, or large variability of I_t . A low error-term is an indication of good fit to data. Different models 'produce' different errors. This is illustrated in graph 5.11.2 for two different models of GDP, say MOD12⁶⁵ and MOD7. We can clearly see that variability of the residual based on MOD7 is much larger than the corresponding for MOD12. That means that there are many changes of GDP over time, which cannot be explained by MOD7, MOD7 has been rejected for other reasons; there is residual seasonal variation, i.e. not white noise. The larger noise of MOD7 is the consequence of the application of a model, which is not proper for the time series. The noise-factor is partly caused by 'nature' and partly our ability to 'understand' nature, i.e. to find a proper model. Every NA series has been carefully analysed in terms of proper models as described earlier. As a result, the impact of noise of the SA series is as low as is attainable. Another reason for increased comparability over time is the new working-day correction, which is used since 1999. Before 1999, the X-11-ARIMA program was used. The working-day correction was made by the ratio-method before seasonally adjustment. As discussed earlier, the working-day effect used earlier was clearly biased. The quarterly changes based on SA series were to a large extent caused by changes of the number of working-days according to the calendar. As a result, there was a large variability stemming from 'overcompensating' working-day effects. This imposed nonsense-variation is now eliminated. A summary of causes for low noise of SA GDP will be as follows:

- 1) TRAMO/SEATS minimize the effects of noise of SA series because of optimal filters for the estimation of the seasonal factor within the framework of model-based signal-extraction. If the theoretical model for the series is true, the reduction of noise is more efficient than other SA methods, e.g. X-12-ARIMA with fixed filters.

⁶⁴ See e.g. Monson, H. Hayes (1996).

⁶⁵ This is in fact the 'official' model for GDP, an ARIMA(110)(010).

- 2) There has been of great concern about the choice of a proper model for GDP and a model with low variability within the class of admissible ARIMA-models has been chosen.
- 3) The working-day correction used earlier by SCB was inefficient and produced extra variability of the SA series. This 'extra variation' has now been of eliminated by means of a regression adjustments of the working-day effects.



Graph. 5.11.2 Residuals from SA of GDP with two ARIMA-Models.

5. 12 Improvements of seasonal Adjustment

Seasonal adjustment of national accounts has since 1999 been carried through with high quality in lines with the recommendations made by Eurostat. Every series has been analysed and the choices of the models have been made on statistical well-known and accepted principles⁶⁶. In some cases, there are some users, which are bothered about 'too much reduction of noise'. The elimination of noise or efficient signal-extraction is the foundation of modern information technology in many disciplines. There are standard courses for the treatment and elimination of noise given at the technical high-schools for engineers. When such an efficient methodology is used for economic signals, we all have to get used to a new picture, a new understanding. Because the differences between the earlier used method, X-11-ARIMA and TRAMO/SEATS are very large in terms of variability; the acceptance of the new picture, the lower noise-level, and the new methods may take some time. When this efficient methodology of signal extraction is applied to economic signals, it seems necessary to give the users time to understand and accept the

⁶⁶ With the exception of the 'principle of smoothness'.

new methods and the new picture of the economic situation. More information from SCB on the new methods would certainly be necessary for this adaptation. When models based on SA series are used by analysts, there seems to be very important that such models are estimated and re-examined.⁶⁷ We conclude that seasonal adjustment of the national accounts is of quite good quality according to the EU-recommendations. There are some issues, which should be further investigated. The working-day variable in the regression models are not always significant at low aggregation levels. Seasonal adjustment made by TRAMO/SEATS estimates outliers very efficiently. The knowledge of these outliers is of great interest. In some cases, we cannot reject the hypothesis that there are statistical errors in the sources for the calculations. Statistical surveillance and quality control of the outliers should be systematically made every time seasonally adjustment is carried out⁶⁸. SCB is waiting for the discussion on time consistency and related consistency issues in seasonal adjustment to converge at Eurostat and ECB. SCB can implement time consistency within the new SAS production system. However, some analysis of the consequences from the users point of view have to be made. Finally, SCB must put more effort and money on the information on seasonal adjustment, e.g. on the Webb.

Responses and suggestions on this document can be sent to sven.ohlen@scb.se.

⁶⁷ There are different opinions among researches on the issue of seasonal adjustment. Some does only recommend original data when further model-based analyses of the economy are made. See e.g. Hylleberg (1992).

⁶⁸ It has in fact been introduced in the national accounts. See Appendix 2.

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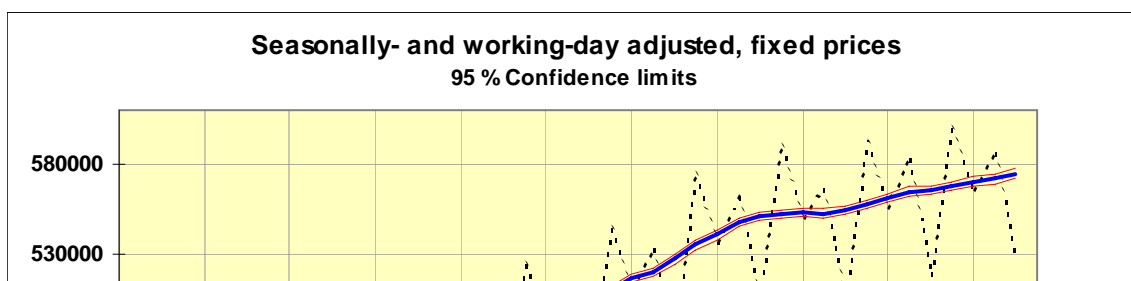
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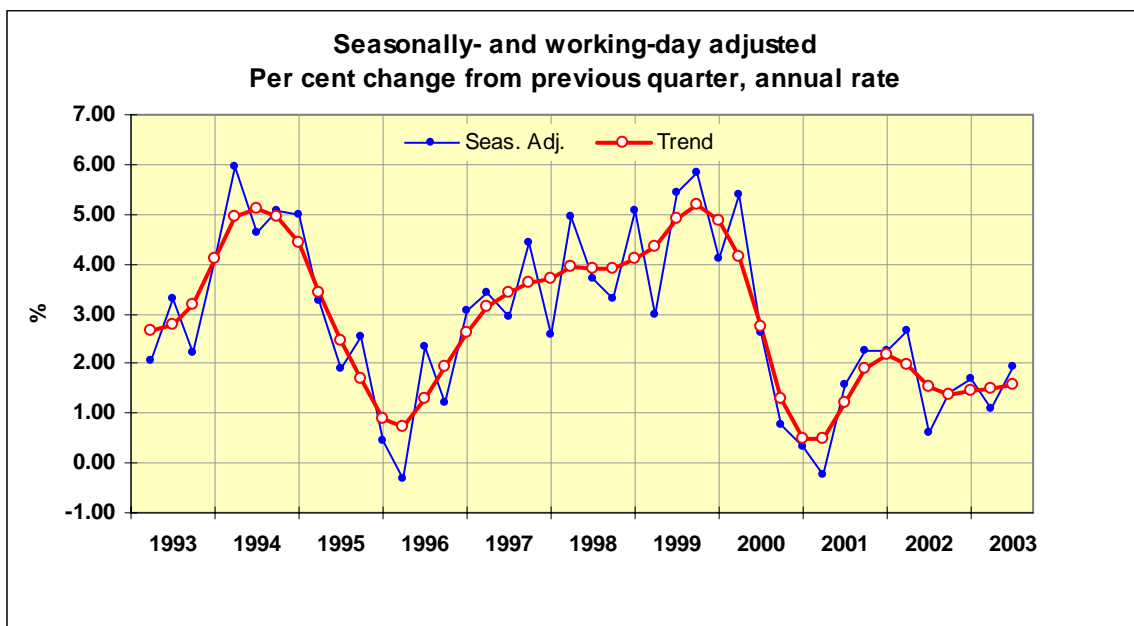
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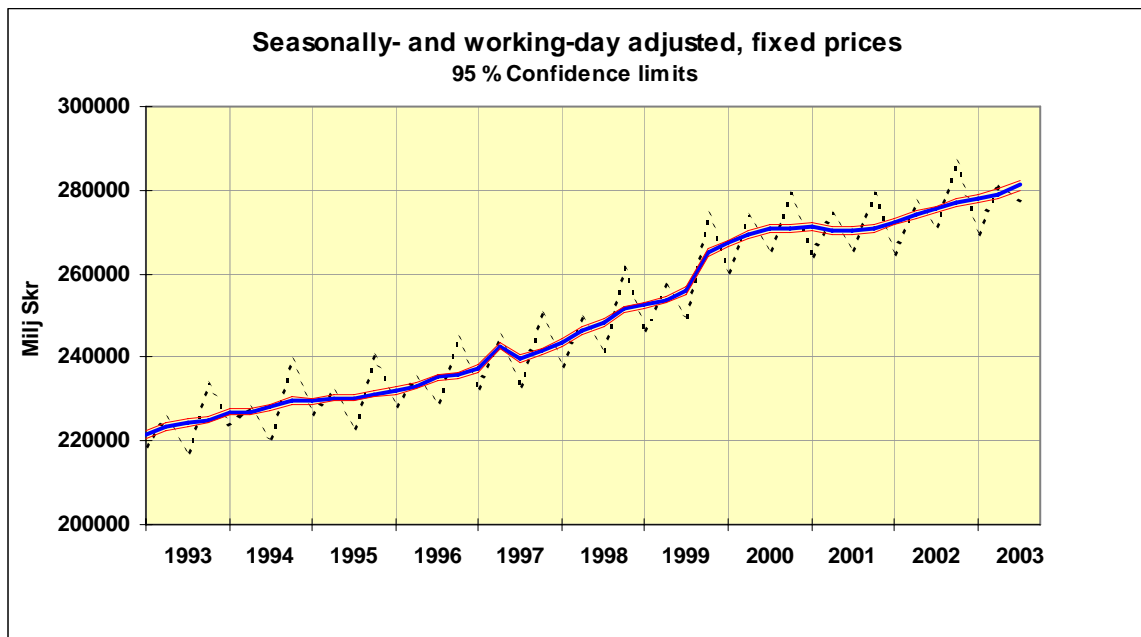
APPENDIX 1. GRAPHS



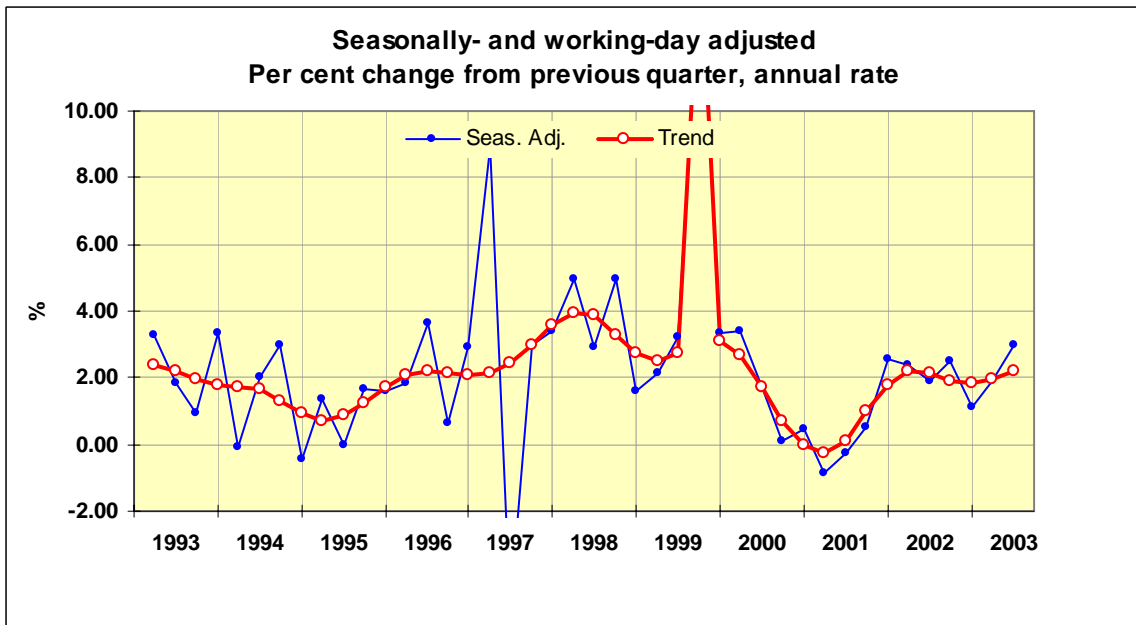
Graph A1. GDP



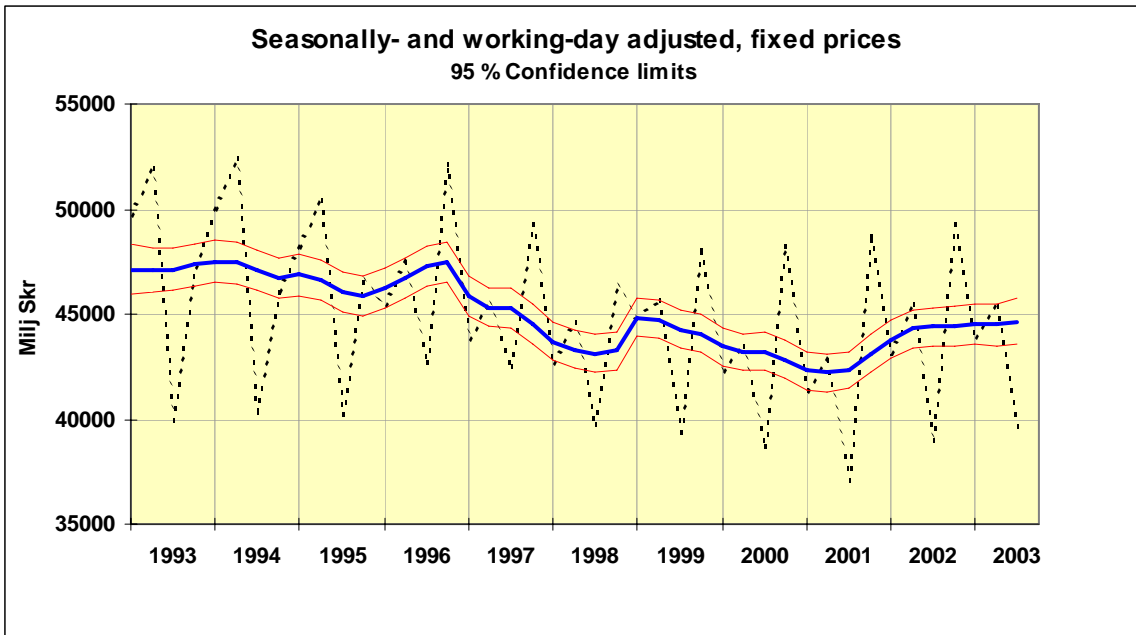
Graph A2. GDP



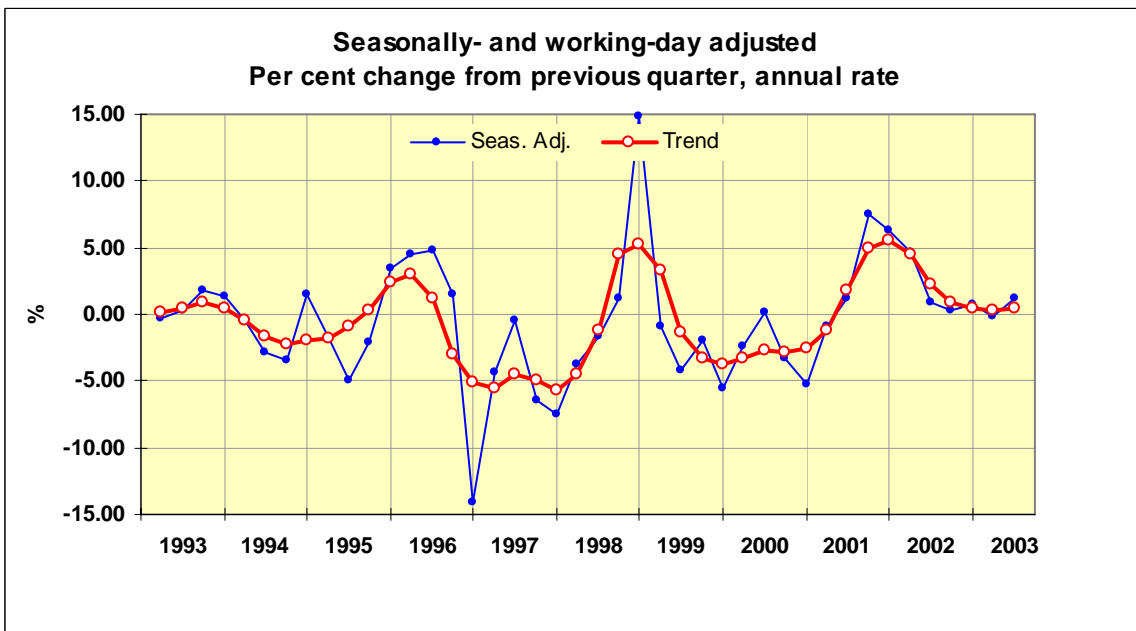
Graph A3. Household Consumption Expenditure



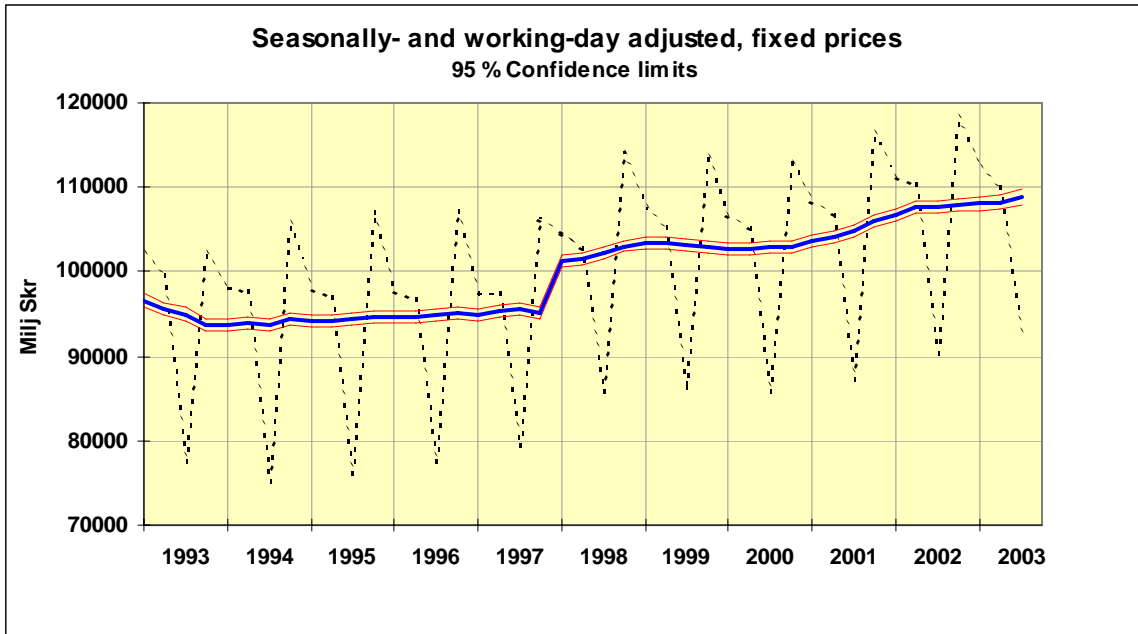
Graph A4. Household Consumption Expenditure



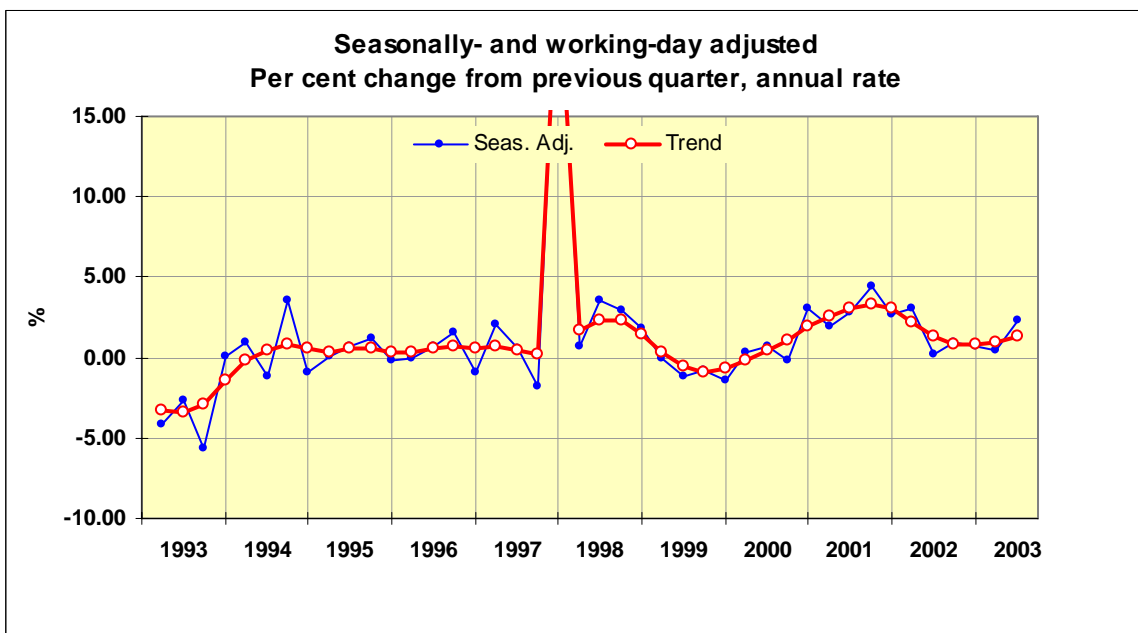
Graph A5. Government Consumption Expenditure



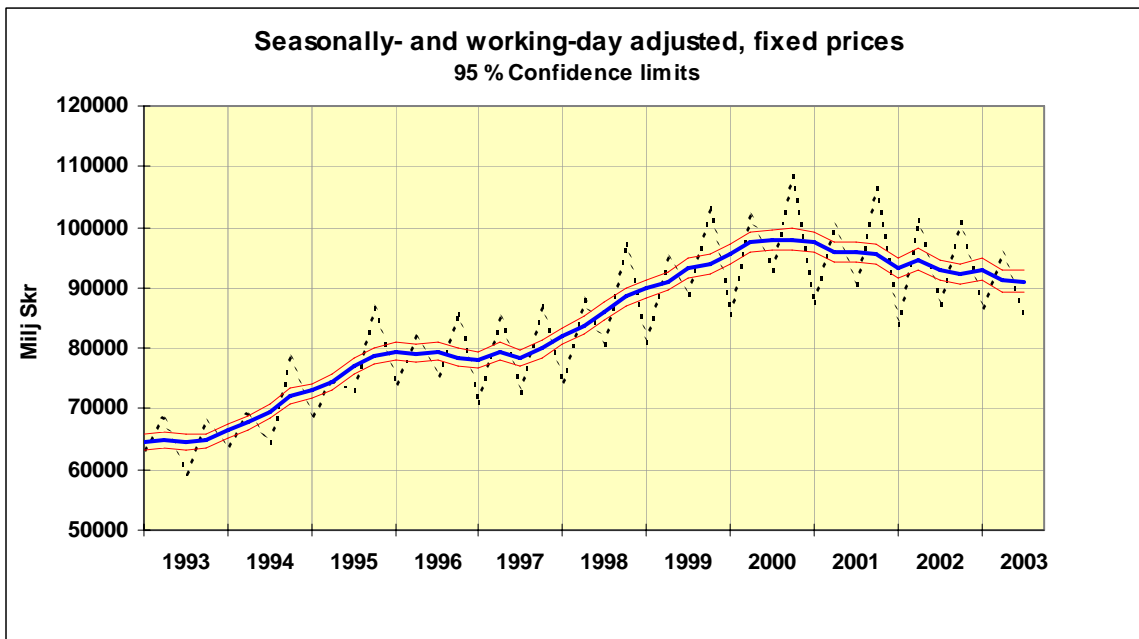
Graph A6. Government Consumption Expenditure



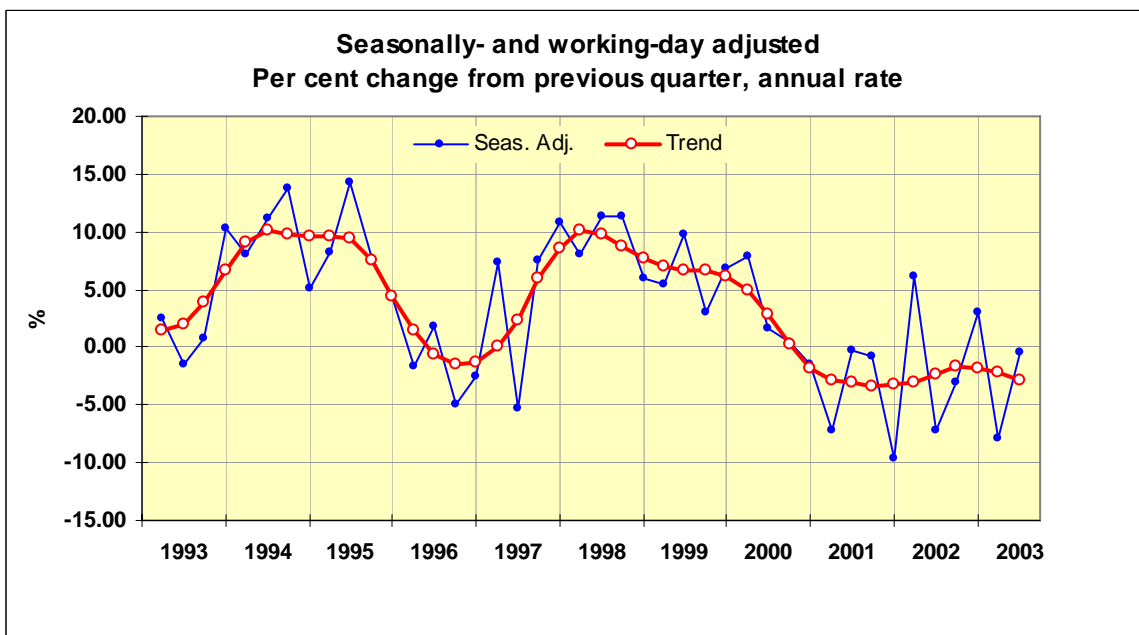
Graph A7. Local Government Consumption Expenditure



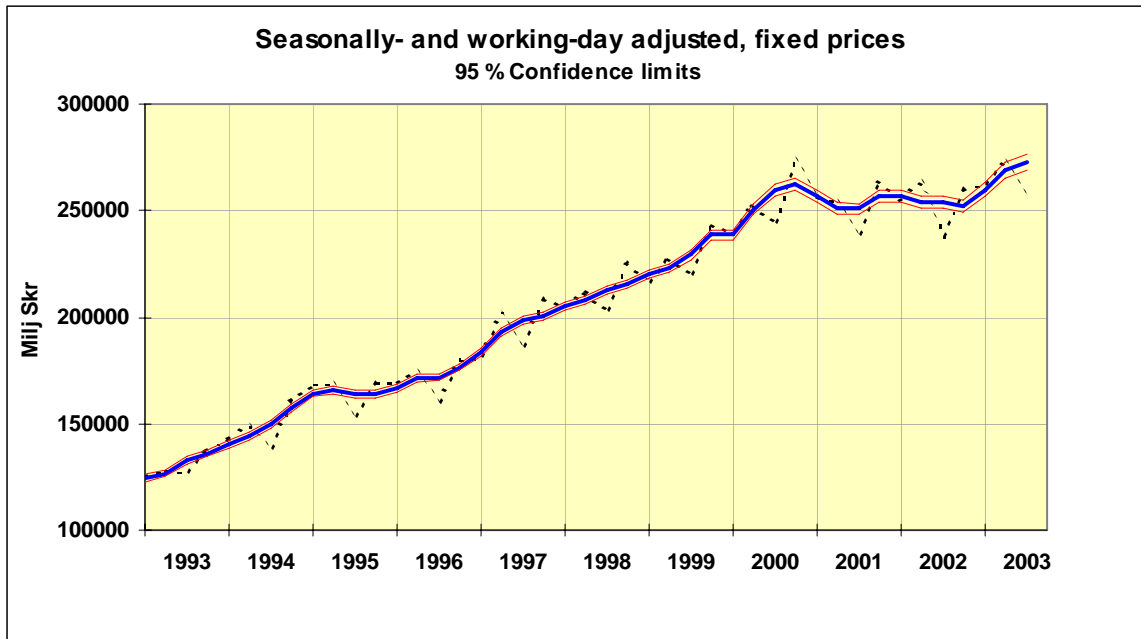
Graph A8. Local Government Consumption Expenditure



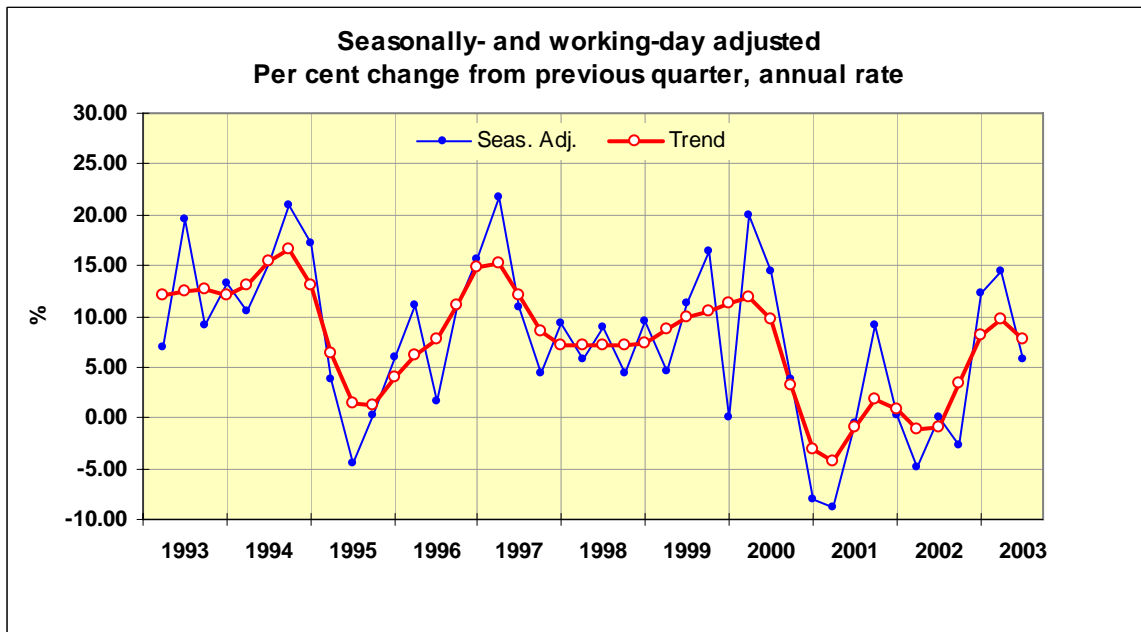
Graph A9. Gross fixed Capital Formation



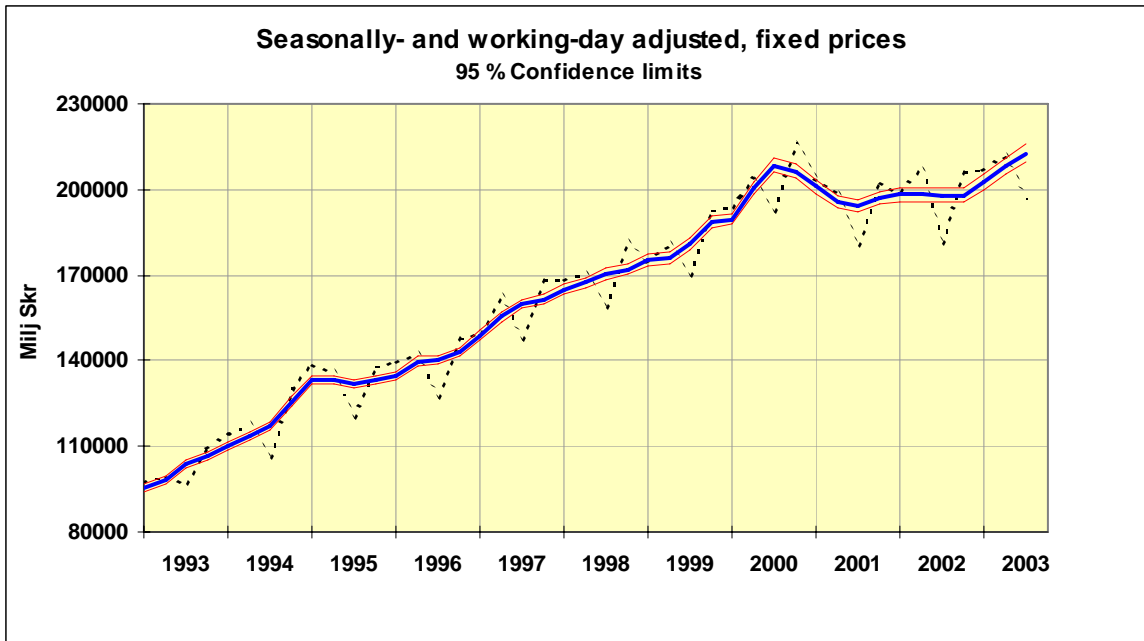
Graph A10. Gross fixed Capital Formation



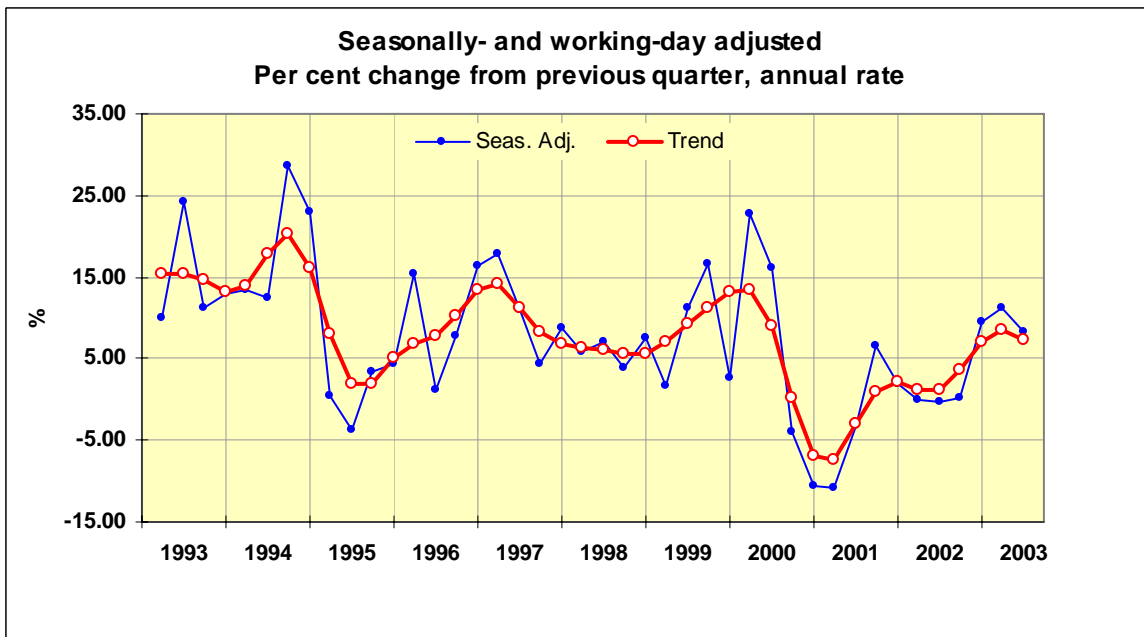
Graph A11. Exports of Goods and Services



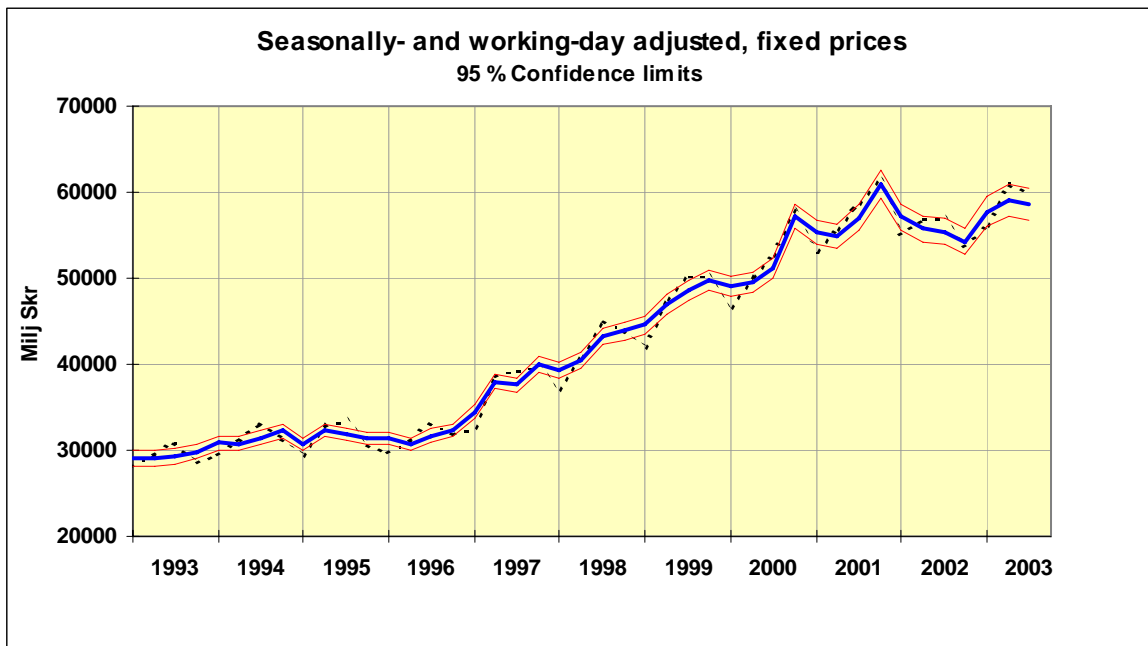
Graph A12 Exports of goods and Services



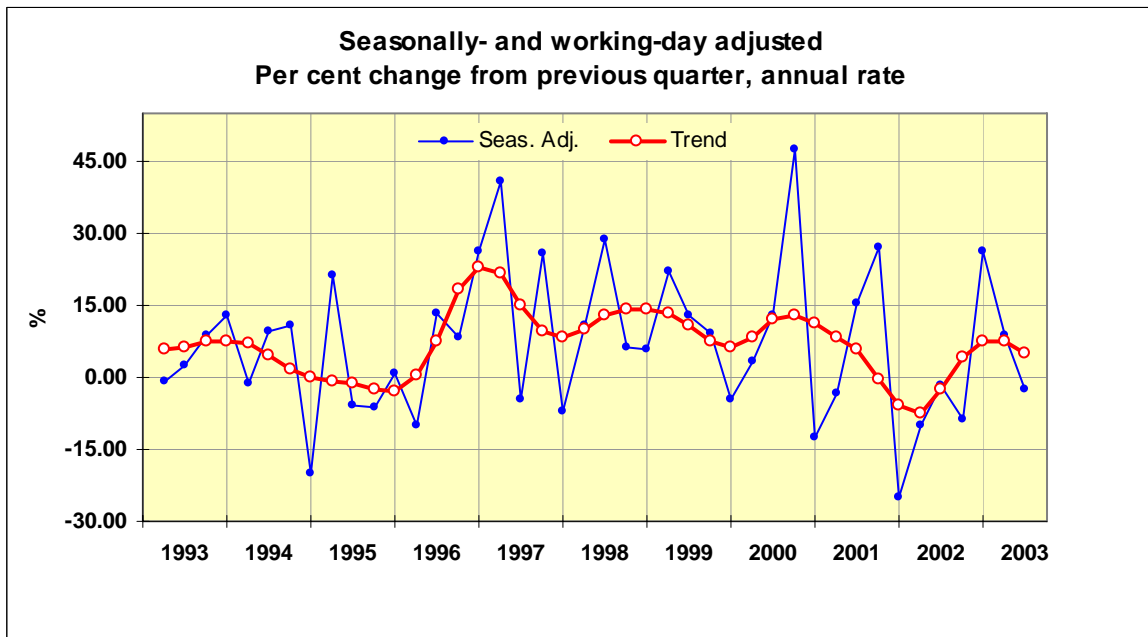
Graph A13 Exports of Goods



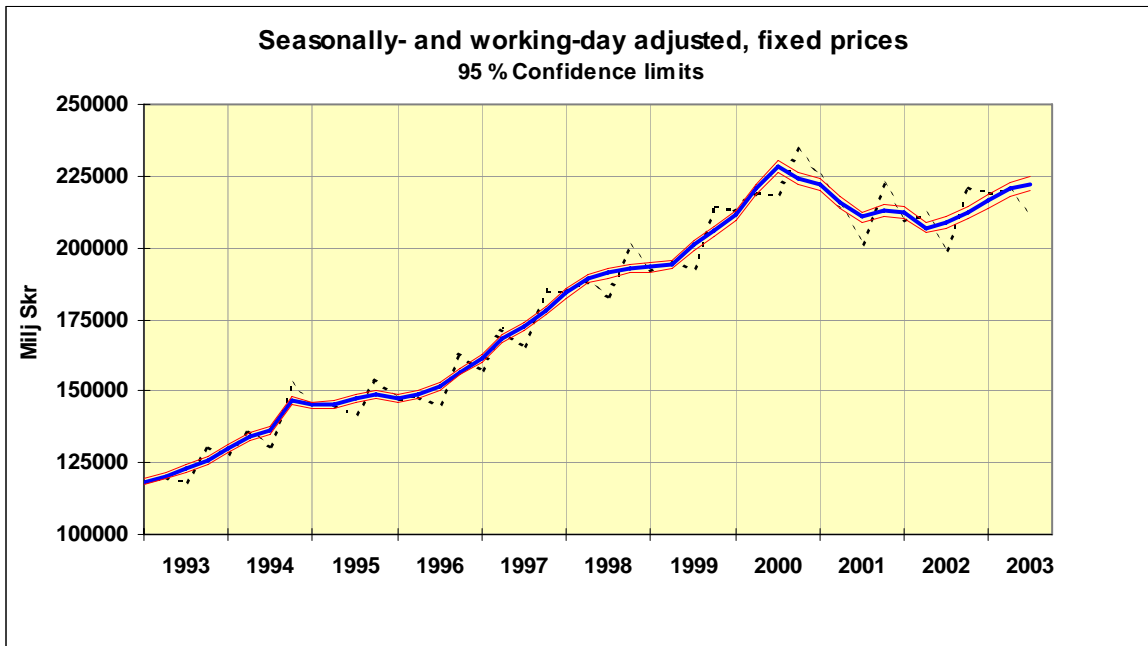
Graph A14 Exports of Goods



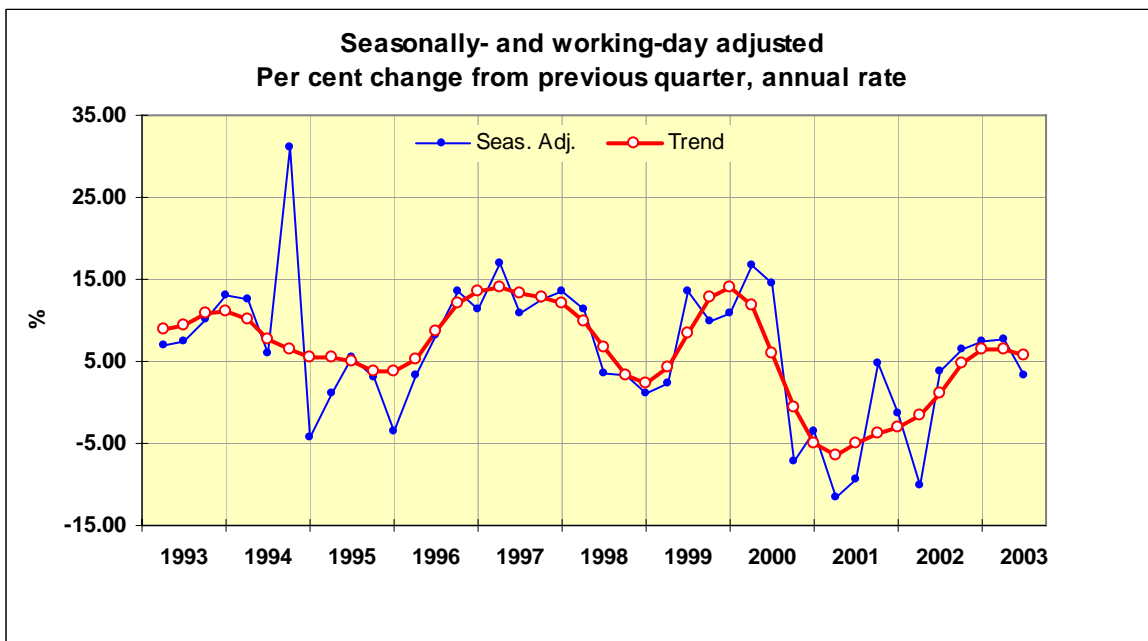
Graph A15 Exports of Services



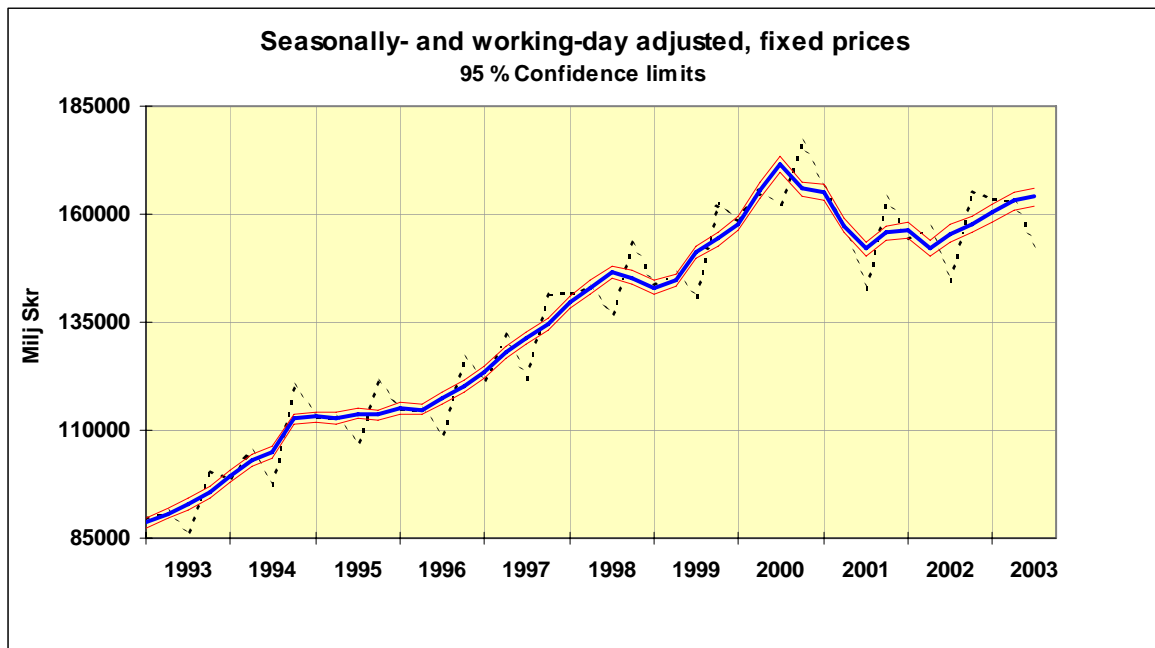
Graph A16 Exports of Services



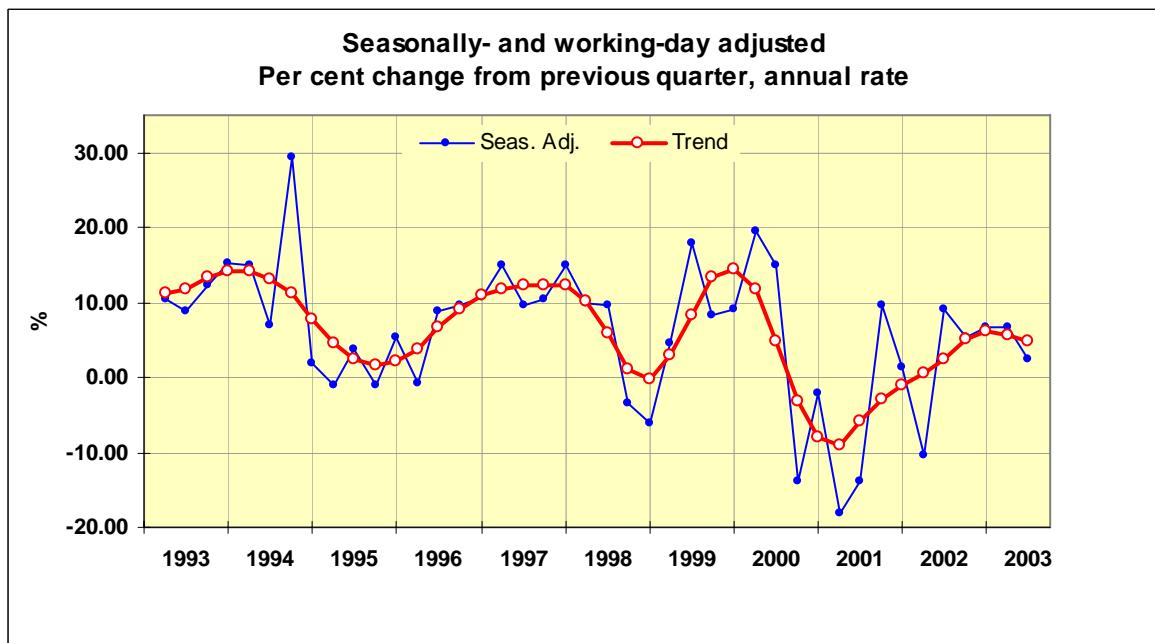
Graph A17 Imports of Goods and Services



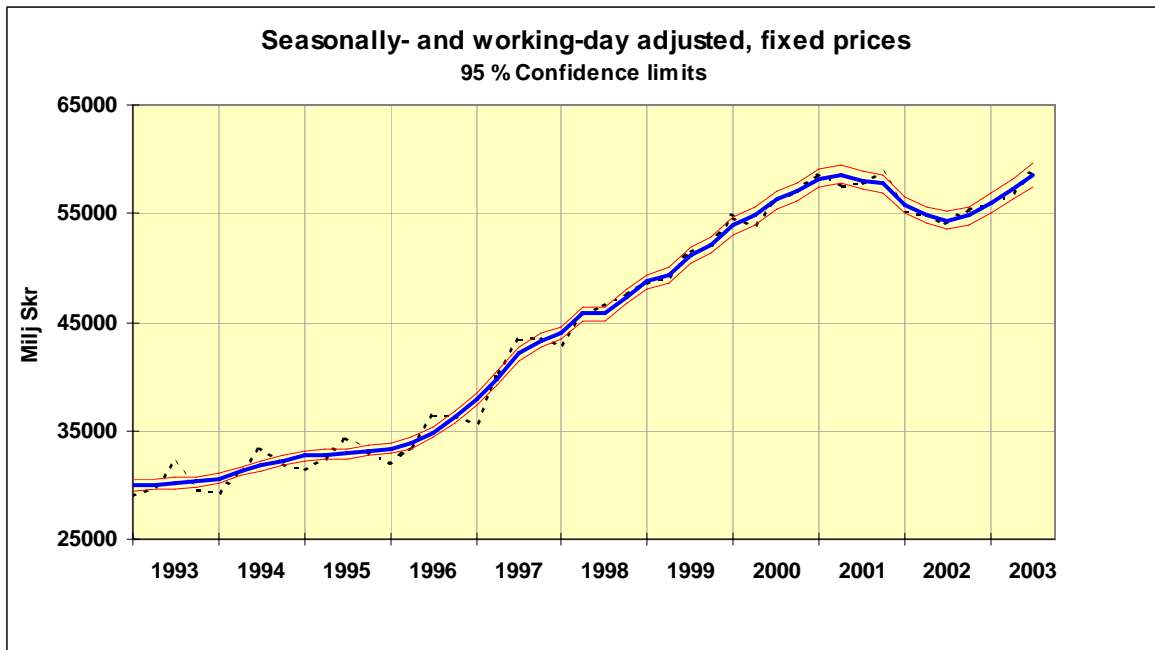
Graph A18 Imports of Goods and Services



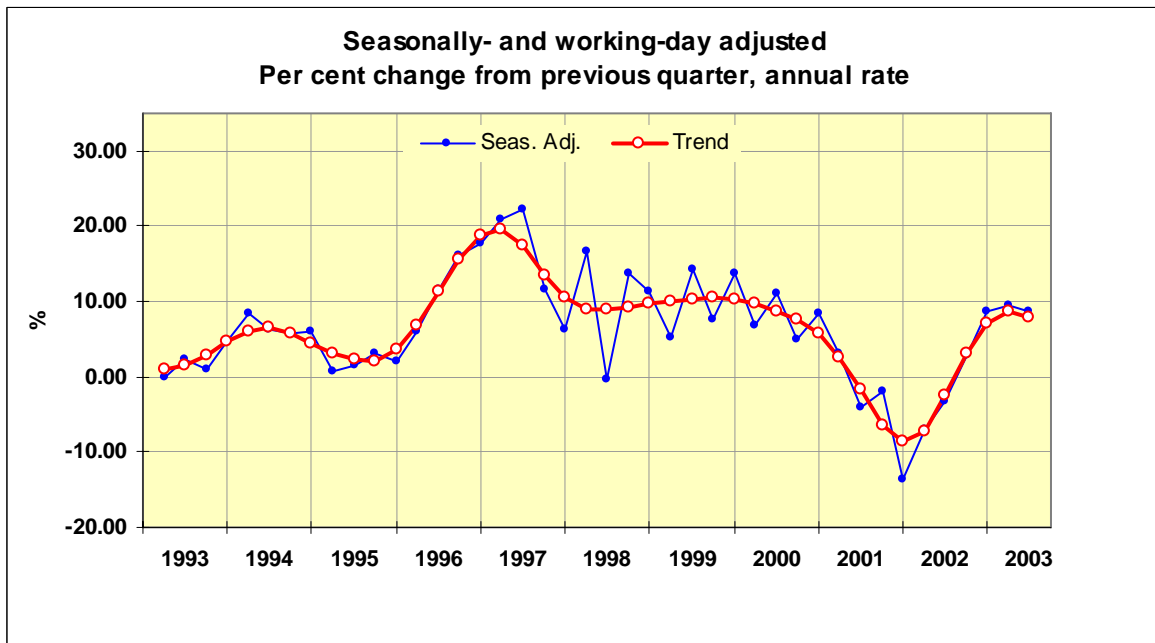
Graph A19 Imports of Goods



Graph A20 Imports of Goods



Graph A21 Imports of Services



Graph A22 Imports of Services